

INTRODUCTION

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THE SECOND PART.

THERE are various applications and definitions given by the Ancients of the word *Musick*; but as they do not fall in with the plan proposed, I shall consider Musick in a less extensive sense, and apply it only to Harmony and Melody.

Harmony is the science of sounds, of which melody is a part.

Air

Air and motion, under certain regular measures and proportions, are the efficient causes.

To discover what this motion is, and how to apply it to musical sounds, so as to be able to know their agreement, difference, or mutual relation, is the object of Science.

It will be proper, as a preliminary, to explain the nature and properties of musical strings, with respect to their different lengths and vibrations, whose agreement or difference is the object of the Ratio.

On these a foundation may be laid, from which a theory, the most rational and agreeable to harmony, may be formed ; one that will not only determine the proportion, agreement, or difference of sounds, with respect to acute and grave, but will teach us their use and application, so as to answer all the demands that harmony or melody can possibly make. But to arrive at this point of perfection, some original leading principle, some universal character must be discovered, whereby not only con-

cords

cords and discords, and their various combinations may be known, but their progressions and successions; for unless this very useful part of musical knowledge can be determined by, and reconciled to, the laws of science as well as of practice, we should still want a faithful guide, an unerring principle.

It is not sufficient to know the several proportions of harmonical combination, we must endeavour to discover some principle that will not only teach us how to form the mass, but how to give it life and motion.

All sounds are formed or generated by motion; that is, by percussion or collision; and as the air is the medium that conveys the sound to the ear, if the motion cease, the sound will also cease; and to whatever distance the sound is carried along this medium, so far the motion passeth, as it were in a sphere; according to the force with which the sonorous body, the centre, is acted upon.

The parts of a sonorous body being thus put in motion, the trembling or vibration is either equal and uniform, or else unequal and irregular; and according to the constitution of the sonorous body is swifter or slower: from whence arises the difference of sounds.

Sounds are concinnous, or inconcinnous; that is, such as are proper, or improper, for music.

Again, they have the difference of acute and grave; which, like great and small, are but relative ideas; each of them has a certain magnitude, but one only is great, and the other small: thus if two sounds of different dimensions be compared, one is acute and the other grave.

This relation of acuteness and gravity is one of the principal and most interesting parts of a musical theory, and is properly the object of mathematical demonstration.

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The degrees of sound may be infinite ; yet with respect to those which are proper for music, they are limited.

All sounds differing in acuteness and gravity constitute an Interval.

This interval may form a concord or a discord, according to the relation of inequality, or particular difference of tune. But in order to fix the degrees of tune, and measure their relations by certain determinate qualities, we must have recourse to the *Vibrations* of elastic bodies and their different *Lengths*.

If two strings of the same length, thickness, and tension, be put in motion, they will vibrate in equal times ; that is, they will move in the same measure, joining and uniting in every course and recourse, keeping still the same equality of motion and identity of tune, having no interval or space between. Therefore the habitude of these two strings is called unison, and has the same relation

to founds that unity has to numbers, and is in the ratio of 1 to 1, or of 2 to 2, &c.

If two strings are in all respects the same, except their lengths, which are in the proportion of 2 to 1, that is, when the grave string 2 is as long again as the acute 1, the gravest found, or longest string, will make but one vibration in the same time that the acute, or shortest string, makes two; and when compared together will make the interval of an octave, whose ratio is as 2 to 1.

If six strings are taken, whose *Lengths* are in the following proportion, 1, 2, 3, 4, 5, 6, the next of any two next numbers will be the acute term, and the other the grave. But it will be the reverse of this if the same numbers are applied to the *Vibrations*; for then the less of any two next numbers will be the grave, and the other will be the acute.

Hence we may observe, that the different lengths of chords are inverse or reciprocal, as their different
vibra-

vibrations ; and we may equally apply to one what is found true or agreeable to the other. For as their lengths are increased, so their vibrations are in the same proportion decreased, and vice versa.

Here then we discover *two distinct characters* in musical strings : the first gives the ratios, which express the intervals from the acute term to the grave, by comparing a shorter string with a longer ; the other gives the ratios, by comparing the vibrations of a grave with an acute sound ; and each of these operations begins at unity, and moves in a contrary direction.

As the ratios are the same in each, it may seem very immaterial whether we descend or ascend from a given sound or pitch ; but when it shall appear that the formation *of the minor and major scales* depends on these two opposite properties in musical strings, the importance of the application will sufficiently justify the distinction.

For the information of those who are unacquainted with the nature of these operations, I
have

have at Plate X. fig. K, placed six lines, which represent so many strings, whose different lengths from unity shew the different ratios of their intervals. The figures placed on the left of these lines 1, 2, 3, 4, 5, 6, indicate their different lengths; and those on the right shew the musical interval, as an octave, 12th, 15th, &c. with a letter annexed to each, the more readily to discover their relation to unity, whose pitch is fixed at B.

Thus from B to B is an octave in the ratio of 1. 2.

From B to E is a 12th, or compound 5th, in the ratio of 1. 3.

From B to B a 15th, or compound octave, in the ratio of 1. 4.

From B to G a 17th major, or a major 3d, twice compounded, in the ratio of 1. 5.

From B to E a 19th, or a 5th, twice compounded, in the ratio of 1. 6.

If we compare their mutual relations we have
2. 3.

2. 3. a fifth,	3. 4. a fourth,
2. 4. an octave,	3. 5. a sixth major,
2. 5. a tenth, or 3d major,	3. 6. an octave.
2. 6. a compound 5th,	4. 5. a major third,
	4. 6. a fifth,
	5. 6. a minor third,

And if the fourth string had been doubled, we should have had the minor sixth in the ratio of 5. 8, which would have completed the whole number of consonant intervals; but as there are only three letters in the whole combination, I have limited the series to six numbers, in order to preserve the fundamental bass E in the grave, with its harmonics, a minor 3d and a 5th, in the ratio of 6. 5. 4.

By these harmonics, which are formed from the different *Lengths* of strings in an arithmetic series, we have the true original principle of the minor mode, which has, by descending from unity, a *minor* 3d, next the fundamental or gravest sound at E 6.

I shall now examine the *Vibrations* by the same six numbers, at Plate X. fig. L.

These,

These, like the former, proceed from unity, but in a contrary direction; for as the number of vibrations in a given time is expressed by the same series as the lengths of strings 1, 2, 3, 4, 5, 6, the slower vibrations are the grave, and the quicker are the acute.

Thus the string that vibrates once is an octave graver than the string which vibrates twice in the same time 1. 2; and for the same reason two vibrations will be a fifth graver than three in the same time 2. 3.

If we proceed with the vibrations, we have all the simple and compound intervals expressed by the same ratios from grave to acute, as they were before by the different lengths of strings from acute to grave; also the *major* 3d next to the fundamental, F, A, C, or 4, 5, 6; whereas the *Lengths* of strings placed the *minor* 3d next the gravest or fundamental sound, 6, 5, 4.

From these opposite properties in the same musical strings we discover the two species of thirds,
and

and in them the essential principles for the formation of the minor and major scales, which in another place will be more fully explained.

I shall now make some observations on the production of consonant intervals as they are derived from a single vibration ; by which the true character of a fundamental bass and its harmonic powers will be found the great and leading principle of a theory of harmonics.

In examining the several intervals which proceed from unity, we find that notwithstanding their great variety their effects are all directed to one point, in order to mark and distinguish one original, or primary, sound, superior to the others.

To illustrate this I shall take the whole series, as set down in the harmonic tables, Plate XI. &c. which are formed in consequence of the vibrations of strings, and are the foundation of the second diagram, as the different lengths are of the first.

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Here we may observe how a given sound at unity in generating its harmonics strengthens itself, and increases its powers by its octaves; thus 1, 2, octave (after which its fifth 2, 3, exists in its way to its next octave 1, 2, 3, 4, which at the same time generates a fourth 3, 4, after the original sound is reinforced by two octaves) proceeds to the imperfect concord the third major 4, 5, or (which is the same) its seventeenth major 1, 5, in its way to its nineteenth 1, 6, or octave to its fifth 3, 6, and finally finishes the whole creation of harmonics with its third octave, 1, 2, 3, 4, 5, 6, 8.

There are two other imperfect concords, viz. the minor third 5, 6, and the minor sixth 5, 8; but as they are formed by the mutual relations, and not by the primary, they cannot be fundamental basses in a major scale.

In the operations above described it must be owned, that nature seems to proceed with the greatest caution in not permitting the fifth and third to appear until the bass advances to its different octaves; thus acquiring fresh power by their addition, it
preserves

preserves the first impression of a generator or fundamental bass, which otherwise might be weakened and overpowered by hearing the fifth and third too soon.

It appears also that the fifth, the next in degree of perfection as a concord to the octave, is doubled in the series, as 2, 3, and 4, 6; the fourth also (by the mutual relations, not fundamentally) is doubled as another perfect concord 3, 4, and 6, 8. But these fourths are only so many octaves to unity, as 1, 2, 4, 8, and therefore are a great addition to its power.

It appears also, that as neither of the thirds or sixes is doubled, one may conclude this is a sufficient reason for their being distinguished as imperfect concords; and being only once heard in the whole combination, they are prevented from becoming predominant, or from diminishing the natural powers of the fundamental bass.

From these observations the character and perfection of a fundamental bass must be admitted as the original principle of harmonic combination ;

since it contains within its octaves all the consonant intervals, which, when united and sung together, yield one of the most pleasing and perfect combinations that can be produced by harmony. But this character does not absolutely depend on its being accompanied with the whole creation of its harmonics, since they are supposed to exist in as full and ample a manner in F, A, C, or 4, 5, 6, as if the whole series in the harmonic tables (which are comprehended in three letters to each root) had been applied : and into whatever position these harmonics may be transposed, as from F, A, C, to A, C, F, or C, F, A, or to any other form, yet F will be the original generator, though less powerful in proportion as the harmonics are further removed from their original place of gravity.

From what has been observed on this subject, it appears to a demonstration how every fundamental bass generates its own harmonics, either by the different *lengths*, or the *vibrations* of strings.

It now remains to shew the relation and connection that, one fundamental bass has with another,
in

in order to discover the true principles by which the motions of the different harmonics are regulated so as to constitute melody.

The harmonics of a given sound, or fundamental bass, can have no motion, or change, but among themselves, unless the fundamental bass moves to another bass, whose relation to it is such as produces the most agreeable alteration possible among the harmonics.

What this relation is with respect to these basses comes now to be explained.

In the arithmetic series, which has been applied to the different lengths of strings, or their vibrations, we find only *these three primes*, 2, 3, 5; but as the ratio 2, 3, is a more perfect relation than 4, 5, or 5, 6, it must therefore have a preference in directing the motion of the fundamental bass.

As the harmonics have been generated from unity, so the motion of the fundamental basses must be directed from the same point.

Thus

Thus if we take a given sound as a principal generator expressed by unity, and compare it with 3, according to the different lengths of strings, we have 1, 3, or a compound fifth descending; and by a like comparison with their vibrations, we have also a compound fifth ascending from unity. See Plate XII. where C is made the principal generator, and has an immediate communication with its two products, F its fifth below, and G its fifth above.

These are therefore the first and primary motions of the principal generator, and their relations are the most perfect, as they form a geometric series in the ratio of 3; for as the vibrations give 1, 3, ascending, and the lengths of strings give 1, 3, descending, the ratios will be equally expressed by continuing the series, as 1, 3, 9; in which the mean, or second, term is the principal, as unity was before: now if each of these three terms be multiplied by the arithmetic series, or only a part of it, as 4, 5, 6, [Pl. XIII.] or 6, 5, 4, [Pl. XIV.] according to the species of third wanted next the fundamental, we shall have the harmonics of each,
from

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from which the systems of tetrachord and diapason are formed.

Hence it appears that a theory of harmonics depends on the *union* of a *geometric* with an *arithmetic* series. The first directs the most perfect motion of the fundamental bass, as from 3, 1, or 3, 9, and by the other we discover the harmonics of each.

These principles being applied to the lengths and vibrations of strings yield all the intervals proper to form the minor or major scales ; and if the geometric progression be continued in the ratio of 3, as in the harmonic tables, we shall have an increasing series of flat dieses by the different lengths of strings, [Pl. XIV.] and of sharps by their vibrations, [Pl. XIII.] together with that vast variety of intervals proper for every mode and gender, and for the mutations.

But before I proceed, it will be proper to take notice of some few operations of numbers that they may be more readily applied to musical intervals.

As

As every musical sound stands in some relation to a given one, or unity, it follows that each must be expressed by its particular number in the harmonic tables, in order to shew its relation to unity, or to any other number, or sound.

There are two sorts, or species, of motion in musical sounds mentioned by the Ancients.

The first the diastematic, or moving by skip, which seems natural to the gravity of the fundamental bass.

The other the systematic, which confined the sounds to the natural degrees of voice, peculiar to melody.

To preserve these species of motion distinct, two sorts of progression are required, which, as they are essential to this theory, are formed into harmonic tables.

The first is the geometric, which is a continued multiplication from unity in the ratio of 3 ;

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as 1, 3, 9, 27, 81, &c. These are placed in the margin on the left of each table, and stand as so many Roots, from which the harmonics are formed by the application of the second progression, that is, the arithmetic; which is placed in every column next the root in this series 1, 2, 3, 4, 5, 6, whose common difference is 1. See Plates XIII and XIV. To this progression the 8th is added, in order to extend the whole to the third diapason, which comprehends, as has been observed before, the whole variety of consonant intervals.

The musical letters stand in the next column, as harmonics to the root; and their relation to each other, or to the root, is determined by the arithmetic series, which also expresses the ratio of each interval.

Each root of the geometric series is the multiplier of the arithmetic, and is the common difference of that series: thus the root 3 is the common difference in this series, 3, 6, 9, 12, 15, 18, the harmonics to that root, which are placed immediately after the letters. Also root 9 will multiply

Q

multiply the arithmetic series, and discover its harmonics, as follows: 9, 18, 27, 36, 45, 54. Hence we not only have their mutual relations, but those which arise from comparing them with the harmonics of any other root, however remote.

The harmonics of each root, as thus described, are in their lowest denomination, and the numbers parallel in the succeeding columns are their respective octaves, made so by multiplying the first, and every succeeding, product of the root by 2.

Thus by multiplying either of the primes 2, 3, 5, by 2, the product will be octave; and if continued, as in these tables, will be a series of geometric proportionals in the ratio of 2: but as a series of octaves, applied as basses or roots, can make no alteration in the harmonics, which will always be the same, the other two primes 3 and 5 will abundantly supply all that is necessary.

We have already seen the great variety which arises from a progression in the ratio of 3; it remains now to proceed to the other, which is also
geometric,

geometric, in the ratio of 5 ; thus, 5, 25, 125. This series is also formed into harmonic tables ; the first at Pl. XV, represents the different lengths of strings, and corresponds with the first diagram : the second agrees with the vibrations Pl. XVI, and corresponds with the second diagram.

In these tables every root is multiplied by the arithmetic series, 6, 5, 4, or 4, 5, 6, by which we have a minor third 6, 5, next the fundamental bass in the first, and a major third 4, 5, in the second diagram ; and by adding 24, a multiple of root 3, taken from the triple progression, Pl. XVI, we have two species of thirds in the harmonics of root 5, in each diagram ; and by proceeding in the same manner with the quintuple progression, we have the same two species of thirds to every succeeding root ; and the semitone minor 24, 25, is not only the interval formed by these two species, but is also discovered in the quintuple progression, between E 120 of root 5, and E# 125 of root 25, in the second diagram.

Q 2.

As

As it is of the greatest consequence in a theory of harmonics to know the original of all intervals which proceed from unity, the better to discover their several relations, I shall mention one more : this is the enharmonic diesis, which is formed by comparing 128, a multiple of root 1, Pl. XIII, with 125, root 25, in the quintuple progression, Pl. XVII.

In these harmonic tables the quintuple progression proceeds from 5, to 25, the next page parallel with it, and after to root 125, in the next or following page parallel with the former ; and in the same manner the roots proceed from left to right, as 15, 75, 375, quintuple, or in the ratio of 5. But if they proceed from one root to the next in the same column, the progression is in the triple, as 5, 15, 45, in the ratio of 3.

It must be observed, that the arithmetic series, which multiplies the root, expresses at the same time the ratio of the products : thus, if we multiply 15 by 6, 5, 4, we have 90, 75, 60, which are in the ratio of 6, 5, 4.

Having

Having explained the principles on which these harmonic tables are constructed, it remains to shew the use of them in a theory of harmonics, by the following rules.

Rules relating to the RATIO.

The harmonic tables contain a series, or progression, of roots from unity in the ratios of 3 and 5, and the harmonics to each are formed by the arithmetic series before described.

Rule I. Every root and its harmonics are multiplied by 2, by which we have a series of simple and compounded octaves continued in the ratio of 2.

II. Hence it follows, that every compounded number in the ratio of 2 may be reduced to its lowest denomination by a continued division by 2 ; and the compounds of the roots of 3 or 5 may be reduced in like manner by a division by 3 or 5.

III. The difference between one number, or term, and another, is the ratio, or difference, between
between

tween one found and another expressed by those numbers.

IV. If one term be compared with another in large numbers, they may be reduced to a simple ratio, by being placed as fractions and divided by a common divisor; thus $\frac{32}{50} = \frac{16}{25}$ the common divisor will be one of the primes 2, 3, 5. In some cases the less term is subtracted from the greater, and the product will be the divisor; thus $\frac{75}{50} = \frac{3}{2}$ a fifth, or $\frac{48}{45} = \frac{16}{15}$ semitone major.

V. When two or more ratios are added together, they must be placed as fractions, and the numerator of the first, and the denominator of the second, must be divided by a common divisor, till brought to their lowest denomination. Thus if we add a tone major and minor together, the sum will

be a major third: $1 \frac{9}{8} + \frac{10}{9} = \frac{5}{4}$; also $1 \frac{5}{4} + \frac{6}{5} + \frac{8}{6} = \frac{2}{1}$.

Thus the major and minor thirds being added to a fourth, the extreme sounds are octave to each other. Also a comma 81. 80, added to a
tone

tone minor 10. 9, is a tone major, $1 \frac{9}{80} + \frac{1}{9} = \frac{9}{8}$.

VI. It happens sometimes that we have no common divisor, in which case the two numerators, and the two denominators, must be multiplied into each other, and if the product has no common divisor, we are sure it is in its lowest denomination.

Thus a major third, and a tone major, added together, $1 \frac{5}{4} + \frac{9}{8} = \frac{45}{32}$, form a greater 4th, or tritonus.

VII. If a minor third be added to a deficient third, we have a common divisor only to one nu-

merator and one denominator, $1 \frac{6}{5} + \frac{32}{27}$. Here 6

and 27 are divided by 3, which gives a new numerator and denominator; but as the division can go no farther, the two denominators must be multiplied for a new denominator, and the two numerators, in like manner, for a new numerator, and the product will be a semidiapente, or im-

perfect 5th. Thus, $1 \frac{6}{5} + \frac{32}{27} = \frac{64}{45}$.

VIII,

VIII. If, after the division is finished, there remain two quotients above, and but one below, or the contrary, the two quotients must be multiplied by each other, and placed as a fraction with the one : thus, if a tone major be added to a semitone

major, $1 \frac{3}{8} + \frac{2}{15} = \frac{6}{5}$, we have a minor third for the product.

IX. In adding intervals together, the numerator of one and the denominator of the other, that are alike, are to be cancelled ; or, which is the same thing, we must write unity over and under each, and then proceed with the division as before.

Thus $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$; then $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$;

then $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$; then $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$;

then $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8}$; then $1 \frac{3}{8} + \frac{16}{15} + \frac{10}{9} + \frac{9}{8} = \frac{3}{2}$,

a fifth, or diapente.

This

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This last operation includes the whole, which, had it not been to shew the progress of the division, would at first have been presented in this form only.

X. If we would know the intervals which compose the system of diapason, we must extract them from the harmonics of any three next terms or roots ; as 1, 3, 9, or 3, 9, 27. The octave of the middle or second term, or principal fundamental bass, is to be placed first, and the succeeding sounds to follow in the order of a system ; after which we must divide each with the next, by a common divisor, or by their difference, and we shall have the ratio of each interval, from acute to grave ; whose sum, when added together, is equal to the diapason.

Example : from 1, 3, 9, of the first diagram, Rule IV.

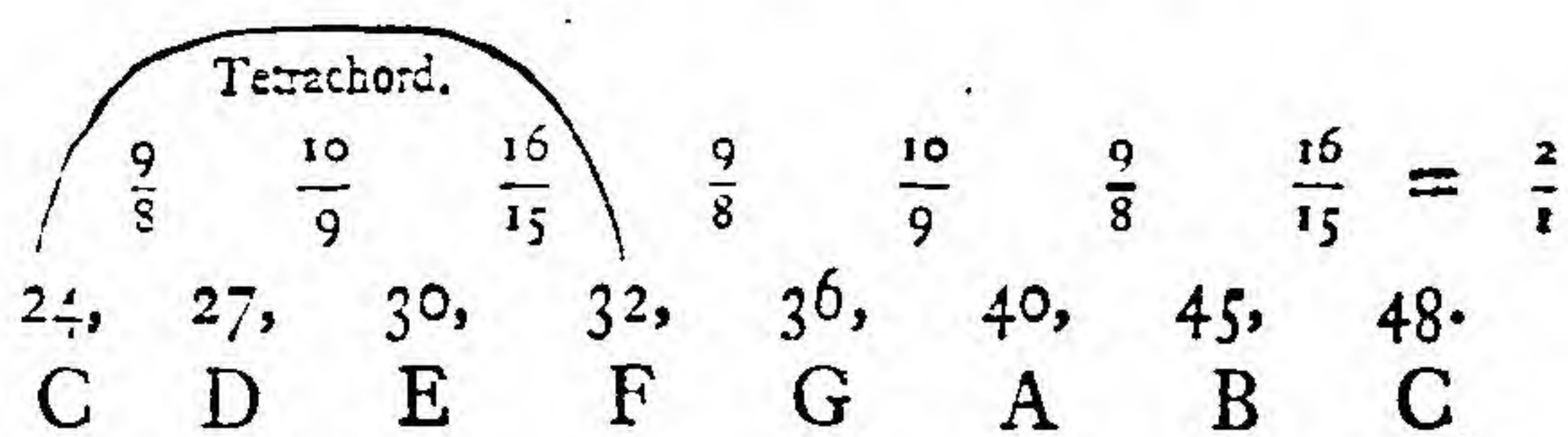
				Tetrachord.				
$\frac{9}{10}$	$\frac{8}{9}$	$\frac{15}{16}$		$\frac{8}{9}$	$\frac{9}{10}$	$\frac{15}{16}$	$\frac{8}{9}$	$= \frac{1}{2}$
36,	40,	45,	48,	54,	60,	64,	72.	
A	G	F	E	D	C	B	A	

R

We

We have here the ratio of $36 \div 40 = \frac{9}{10}$ a tone minor, and $40 \div 45 = \frac{8}{9}$ a tone major; and of the others, placed in fractions above, in this example.

XI. It must be observed, that in a descending series from acute to grave the fraction is formed by placing the less number above, but in an ascending series the less number of the fraction is below; as in the following example, taken from the three first terms of the second diagram, 1, 3, 9.



In these two examples of diapason the great advantage of the ratio is discovered; for though each is composed of three tones major, two minor, and two semitones major, yet their position in each is very different; and as the harmonious constitution of the tetrachord, which is the product of the three next terms, 1, 3, 9, or 3, 9, 27, &c. is the original of these and all other scales, we must look there for their essential differences.

Thus,

Thus, in the first tetrachord from C to D is a tone minor, which in the second is a tone major: also from D to E is a tone major in the first, and a tone minor in the second. Besides this, as the intervals in each diapason are the products of the series 1, 3, 9, which is in the ratio of 3, their motion or succession must be conformable to that series, which will not permit us to move from 40 to 45, because they are the products of 1, 9, and not 1, 3, or 3, 9, which regulate the motion of all the others.

From these observations we discover that the *geometrical* progression of the fundamental bass is the *governing principle*, and that the system of tetrachord, which made so great a part in the Grecian theories, had, from its simplicity, a preference to the diapason, which, when applied to the fundamental bass, or three terms, is incapable of proceeding as a system from one extreme to the other, unless the system is discontinued by an incomposite interval of a third, as in the following example, which is the same descending or ascending.

R 2

C D

$$\left\{ \begin{array}{cccccccccc} \text{C} & \text{D} & \text{E} & \text{F} & \text{G} & \text{A} & \text{C} & \text{B} & \text{C.} \\ 24, & 27, & 30, & 32, & 36, & 40, & 48, & 45, & 48. \\ 3, & 9, & 3, & 1, & 3, & 1, & 3, & 9, & 3, \\ \text{C} & \text{G} & \text{C} & \text{F} & \text{C} & \text{F} & \text{C} & \text{G} & \text{C.} \end{array} \right\}$$

In this progression it appears how great a share the second term 3, or principal generator, has in respect to the first and third terms 1 and 9.

XII. To know the relation of a root and either of its harmonics, they must be brought as near together as possible, by dividing the harmonic by 2. Thus, to find the ratio of 27. 9; as I cannot divide 27 by 2, I must multiply 9 by 2, which gives 18, and then (by Rule IV.) $\frac{27}{18} = \frac{3}{2}$, also of 49. 9; 9 must be multiplied till brought the nearest to 45, as $\frac{45}{36} = \frac{5}{4}$, a major third.

But in order to save the trouble of multiplying or dividing the terms to be compared, we may extract them from the harmonic tables.

XIII. Thus

XIII. Thus if we compare the root 3 and 729, in the columns parallel with 3, we look for one of its multiples that comes the nearest to 729, and find it to be 768; then placing them as fractions, and proceeding by Rule IV. $\frac{768}{729} = \frac{256}{243}$, we have a limma, or the difference between C in a natural scale, and B in a scale with four sharps. Also if 81, a fifth to root 27, be compared to 5, a major third of root 1, we proceed as before, except in the division, and find the ratio $\frac{81}{80}$ a comma, which is the difference of A in C scale, and A in G scale; and if we add the comma to the limma, by Rule V, we have a semitone major: thus

$$\frac{256}{243} + \frac{81}{80} = \frac{16}{15}.$$

In the same manner we may proceed to discover by these tables all other intervals, however remote: as for instance, the *Pythagorean* comma 524288, and 531441, which is a comparison of a multiple of the first root, with the fifth of the twelfth power, root 177147.

It must be particularly observed, that every scale is perfected by the harmonics of the three
next

next terms, and that the product of no other can enter into its composition: thus, 1, 3, 9, or 3, 9, 27, &c.

XIV. If we would know the difference between the first and last note of a series, we must observe that the ratio is formed so as to distinguish the ascending or descending intervals, by placing the greater or less number of the fraction uppermost; agreeably to Rule XI; after which we may proceed to add them together, according to Rule IX.

C G A E F C G

Thus, $1 \frac{3}{4} + \frac{10}{9} + \frac{3}{4} + \frac{16}{15} + \frac{3}{4} + \frac{3}{2} = \frac{3}{4}$.

In this example the last note is a fourth lower than the first.

These rules being sufficient for the *addition* of intervals, I proceed to explain how a less interval may be *subtracted* from a greater, in order to discover the difference.

XV. In comparing one interval with another they must be placed as fractions, and the sign of multiplication

multiplication put between them, to shew that the numerator of the first is to be multiplied by the denominator of the second ; and the denominator of the first by the numerator of the second. Thus the difference of a tone major when compared with a tone minor is a comma : $\frac{9}{8} \times \frac{10}{9} = \frac{81}{80}$, a comma. between an octave and fifth, $\frac{2}{1} \times \frac{3}{2} = \frac{4}{3}$, a fourth ; between a fifth and fourth, $\frac{3}{2} \times \frac{4}{3} = \frac{9}{8}$, a tone major ; between a fifth and sixth major, $\frac{3}{2} \times \frac{5}{3} = \frac{10}{9}$, a tone minor ; between a sixth minor and a fifth, $\frac{8}{5} \times \frac{3}{2} = \frac{16}{15}$, a semitone major.

If the products are not in their lowest denomination, after this cross multiplication of the ratios is finished, they must be divided by a common divisor. Thus, if from a fifth we subtract a major third, the remainder is a minor third : $\frac{3}{2} \times \frac{5}{4} = \frac{15}{8} = \frac{150}{80} = \frac{15}{8}$, a minor third.

If we subtract a semitone major from a tone minor, we have a semitone minor : $\frac{10}{9} \times \frac{16}{15} = \frac{160}{135} = \frac{32}{27}$. The common divisor here is 6, the difference between

tween 150 and 144, (see Rule IV.) and the product is a semitone minor.

If from a minor third a deficient third is taken, the residue is a comma : $\frac{6}{5} \times \frac{32}{27} = \frac{162}{160} = \frac{81}{80}$, a comma.

XVII. When the multiplication of the ratios is finished, and the products have no common divisor, we may be sure they are in their lowest denomination : as for example, the difference between the tritone 45. 32, and the semi-diapente 64. 45, $(\frac{45}{32} \times \frac{64}{45})$ is $= \frac{2048}{2025}$, their lowest denomination.

XVIII. If the ratio, or difference, is not well understood, we must look for the numbers in the harmonic tables in either of the diagrams : as for example, 2048 is a multiple of root 1 B, in the first, or of F, root 1, in the second diagram ; and 2025 is Cb, a minor third to the fundamental bass, root 405, in the quintuple of the first, or is E#, a major third to the fundamental bass at root 405, in the quintuple of the second, diagram. If, after this, a more satisfactory knowledge of this ratio is wanted, we may add it to some other small quantity

tity as at Rule V. For example, $\frac{2048}{2025} + \frac{81}{80} = \frac{128}{125}$, an enharmonic diesis.

Also a semitone minor added, $\frac{2048}{2025} + \frac{25}{24} = \frac{256}{243}$, a limma. Or a greater semitone minor added, $\frac{2048}{2025} + \frac{135}{128} = \frac{16}{15}$, a semitone major.

To account for these two species of minor semitones, as well as the two species of major, we must subtract either a semitone major, or a semitone minor, from a tone major, and the residue will be in the first case a greater semitone minor, and in the second a greater semitone major.

For example, $\frac{9}{8} \times \frac{16}{15} = \frac{135}{128}$ exceeds $\frac{25}{24}$ by a comma; and $\frac{9}{8} \times \frac{25}{24} = \frac{27}{25}$ exceeds $\frac{16}{15}$ by a comma.

From these divisions of the tone major, the impossibility of an *equal* division of intervals is very evident; for if we divide the tone minor, it will be also into unequal parts. Example, $\frac{10}{9} \times \frac{16}{15} = \frac{25}{24}$, a semitone minor. And the same inequality will be found in dividing all other intervals discovered in a theory of harmonics.

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There are other methods by which the addition and subtraction of intervals might have been explained; but as these appeared to be the least difficult, I have therefore given them a preference, and doubt not but they will be found sufficient when applied to the harmonic tables; in which, every interval, however minute, may be discovered, as also its root, or fundamental bass, together with its scale and relation to all others, whether with minor or major thirds; for I must observe, that there are as many positions of these two species of scales, as there are flat or sharp dièses, exclusive of the two natural scales.

These are the principal *Rules* in respect to the *Ratio*, by which the following theory will be understood without much difficulty.

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THE
THEORY
OF
HARMONICS.

PART THE SECOND.

SECTION I.

AMONG the works of the Grecians, published by M. Meibomius, we have an Introduction to Music by Guarentius, the Philosopher, a favourer of the Ratio. In the 17th page of his Introduction, he has laid down *two Diagrams*, which, being in their degrees composed of tones major and limmas, have given occasion to the most formidable objection to the Grecian Theory.

These two diagrams I come now to examine. They are to be considered as *one general and universal system*, in which all particular systems are contained.

A particular system may be limited to some part of an universal one, but is always a regular collection, or composition, of many things; by which a chain of principles is linked together, and their several parts follow, and depend on each other.

In explaining these diagrams there are two things to be kept constantly in view: the one, the different *lengths* of strings, which is the foundation of the first diagram; and the other, their different *vibrations*, which is the foundation of the second.

These diagrams, as they now stand, are composed of tones major and limmas, which are placed before the large numbers in the first, and after them in the second, diagram. See Pl. XVIII. The fifteen names are common to each; but the numbers placed before the names represent the lengths of strings in the first diagram, and those placed after represent the vibrations in the second.

But in order to discover the original series of which these numbers are composed, they must be divided by 2, to bring them to their lowest denomination.

$\frac{9}{8}$	$\frac{9}{8}$	$\frac{256}{243}$	$\frac{9}{8}$	$\frac{256}{243}$	$\frac{9}{8}$
2592	2304	2048	1944	1728	1536
1296	1152	1024	972	864	768
648	576	512	486	432	384
324	288	256	243	216	192
162	144	128		108	96
81	72	64		54	48
	36	32		27	24
	18	16			12
	9	8			6
		4			3
		2			
		1			

From these reduced numbers a geometrical progression is formed in the ratio of 3; as 1, 3, 9, 27, 81, &c. in the harmonic tables belonging to each diagram. Pl. XIII and XIV.

I shall now place this series in the following order, beginning with unity, and annex to each number one of the seven musical letters; the choice of which is determined by the greatest number of fifths ascending and descending without being obliged to make use of either flats or sharps.

First Diagram.	Lengths of strings descending from unity.					
{ B	E	A	D	G	C	F
{ I	3	9	27	81	243	729
{ F	C	G	D	A	E	B
Second Diagram.	Vibrations of strings ascending from unity.					

From

From this description we may observe, that the different lengths of strings are inverse, or reciprocal, as their different vibrations; that by these properties in musical strings, the series in the first diagram must descend in the ratio of 3, by a necessity equal with that which obliges the second to ascend from unity in the same ratio.

We also discover, that instead of a scale of very untunable degrees, such as are tones major and limmas, we have a continued series of triples, in geometric proportion, each of which being multiplied by an arithmetic series 1, 2, 3, 4, &c. generates its own harmonics and all other consonant intervals, and must therefore be considered as the root, or principal, by which those harmonics exist, as may be seen in the harmonic tables, Pl. xiv, &c.

This progression from each to the next is in a compound ratio of a fifth, viz. a twelfth; by which means we have the octave to 1, in our way to the fifth, 1, 2, 3. This octave appears necessary to every root; not only to approach the fifth in its true ratio 2. 3, but also to unite the two great principles on which this theory so much depends, which are the *geometric* with the *arithmetic* progression.

These octaves necessarily multiply the first series of seven roots into fourteen, when ranged in a scale, as in the two diagrams, Pl. xviii; but as the Grecians deemed this an imperfect system, they added another sound, viz. Proslambanomenos, in the ratio of 1. 4, by which the whole was comprehended in two octaves, when formed into degrees, which
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they reckoned the most perfect system, as containing all the less. See Pl. XIX; in which the two diagrams are inverted, and the musical letters applied to each sound.

That a series, in the ratio of 3, should be formed into a scale of degrees, consisting of tones major and limmas, seems necessary to preserve the numbers according to their magnitude, as they represent the different lengths of strings and their vibrations; to describe both of which properly required the inversion of the scales to be placed in opposition to each other, in order to shew that the same number which represents an acute sound in the first, was the same which represented a grave sound in the second. But as they could not in this form be applied to either of the genders, it cannot be doubted but that some other form was necessarily implied, and generally known to be the only rational foundation of a theory of harmonics.

Nevertheless it must be owned, that for want of a more particular explanation of this doctrine, the writers of succeeding ages have been mistaken, and have formed only one scale of degrees out of the two diagrams; and some writers on this subject have explained grave sounds as high, and acute as low.

The whole theory thus disordered, and every principle of science extinguished, gave occasion to the great objection to the Grecians' having any knowledge of harmony; namely, that the imperfect concords, $\frac{5}{4}$, $\frac{6}{5}$, $\frac{5}{3}$, and $\frac{8}{5}$, which are so essential to harmony, are not to be found in their true ratio.

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This objection must remain unanswerable, if no other application than that of a scale of untunable degrees can be discovered. But the contrary of this has already appeared, by reducing the large numbers to their lowest denomination, by which we have found their real application in a geometric progression in the ratio of 3.

If we examine the two diagrams, (Pl. xviii.) the Proslambanomenos 2304 in the first diagram, is the lowest or gravest found; but the numbers of the Proslambanomenos of the second diagram, as the gravest found, are 648. Also the highest, or Nete hyperbolæon, of the first is 576, whereas the highest of the second is 2592. In the same manner (Pl. xix.) the numbers in the first, to which A or B are annexed, are the same as those of G or F in the second, as D is the only note to which the same numbers are applied, and is of the root 27 in each, by being the same distance from unity in each.

The inversion of the scales, agreeably to the two great principles, makes no alteration among the intervals, which are the same above and below, as appears by the ratios, and by the black notes, which distinguish the limma, as the open notes do the tones major.

These large numbers in the diagrams have already been discovered to be a geometric progression ascending and descending, in the ratio of 3, as 1, 3, 9, &c. and have each been distinguished as a root, when multiplied by the arithmetic series, 1, 2, 3, 4, 5, &c. by which operation we discover all possible consonance belonging to that root, which cannot be applied to
any

any other. As these harmonics are all in their true ratio to the root, as well as among themselves, their relation and agreement with the harmonics of any other root, may very easily be discovered by the rules already laid down for the addition or subtraction of ratios.

It appears in the harmonic tables, Pl. xiv, &c. that the product of each root is comprized in three letters: to discover therefore which of these three letters is the fundamental bass, we must bring them the nearest that is possible to each other, and the gravest or lowest of the three will be the fundamental bass.

Thus in the first diagram of the harmonic tables, 6, 5, 4, are the nearest to each other, and their relations stand thus $\frac{5}{6} + \frac{4}{5} = \frac{2}{3}$, where 6 is the grave sound, and consequently the fundamental bass.

But the contrary of this is true in the tables of the second diagram, where 4, 5, 6, place 4 as the fundamental bass, as $\frac{5}{4} + \frac{6}{5} = \frac{3}{2}$.

Hence we discover that no root in the first diagram can be a fundamental bass, as they all are in the second; for instance, in the first, unity, or B, is the root, yet E 6 is the fundamental bass; but in the second, unity, or F, is the root, and also the fundamental bass, whether it stands as 1, 2, or 4; for these octaves make no difference with respect to its power as a fundamental bass.

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It must be observed, that these differences are occasioned by the two first principles of the lengths of strings, and their vibrations; the first places unity in the acute, and descends to the grave, as the numbers increase; and the other places unity in the grave, and in like manner as the numbers increase ascends to the acute.—To express which, it was necessary to insert the ratios, and place the largest number of the first contrary to what it is in the second example.

This combination 6, 5, 4, or 4, 5, 6, forms the trias harmonica, which being sung together, produces the most perfect union of consonance by their mutual agreement.

But that the roots, the fundamental basses, and their harmonics may be well understood, by those not perfectly acquainted with a theory formed on these principles, Pl. xx and xxi are designed to illustrate them.

Plate xx, comprehends the seven roots of the first diagram, in their lowest denomination, from unity to 729, which are placed in the first column, 1, 3, 9, &c. descending from acute to grave in the ratio of 3.

The three letters in the next column express the trias harmonica, of which the gravest is the fundamental bass, before explained.

In the third column are the numbers, or products of the roots when multiplied by 6, 5, 4, by which a comparison of the harmonics of one root is easily made with those of any other,
by

by the rules before explained for the addition, or subtraction, of intervals.

In the following columns are the multiples of the root by 2, in large numbers, which correspond with those in the first diagram, at Pl. xviii.

Pl. xxi, containing the second diagram, has the same number of columns as the first, and for the same reasons; except instead of descending from acute to grave, it ascends from grave to acute, and the root in this is the fundamental bass, contrary to what it was in the other; and as the progression advances from 27 to 729, the series of sharps is discovered in their proper order; F #, C #, G #, D #: as the flats, B b, E b, A b, D b, are in the first diagram, from 27 to 729. We have also the harmonics for the minor mode or scale in the first diagram, and for the major in the second.

From what has been explained, it appears, that the principles on which this theory is founded are divided into two parts, and that each is of equal importance to the whole; that the ratio of concord, or discord, in the first is the same in the second diagram, but in a contrary direction from unity; that the *minor* scale is not formed *out* of the *major*, but each is *original*, and depends on distinct principles; each has its roots, its fundamental basses, and harmonics, and whatever else is necessary to complete the most perfect theory, by supplying every demand that *Melody* or *Harmony* can make.

SECTION II.

OF THE TETRACHORD.

The Tetrachord, in the Diatonic gender, is a system the most natural, and agreeable to the voice of man. Its composition consists of four sounds, or three intervals, of which the semitone is the first, and is succeeded by the tone minor, and finishes with the tone major, in the first diagram.

These three intervals, being the product of any three next terms, in a geometric progression in the ratio of 3, are fixed and unalterable in the diagram to which they belong; and cannot therefore be considered as a simple melody only, but as a composition of several, whose union with, and dependence on the three terms, or fundamental basses, makes the whole truly harmonical, as appears by the two tetrachords formed in Pl. XXII, fig. 1 and 2, extracted from the two diagrams.

In the first, we have all the sounds proper for the minor scale, and in the second the major is completed.

The lowest space of each tetrachord has the roots, 1, 3, 9, with the fundamental basses in the space above, after which, the three next spaces contain the three melodies, with figures annexed to each, to shew their relation to each other, and to the fundamental bass. The upper of the three spaces is the tetrachord, B, C, D, E, moving from left to right, with the ratio of each interval placed above, as determined by the roots, or fundamental basses; and the whole makes a most agreeable succession

succession of harmonic sounds, either ascending or descending, and the different positions of tones, major and minor, are distinguished according to the scale; thus, from C to D, which is a tone minor in the first, is a tone major in the second; also from D to E, which is a tone major in the first, is a tone minor in the second tetrachord.

It follows next to examine the several parts of this harmonic composition of tetrachord, to see if any thing characteristic can be discovered in the motion of these harmonics, as they appear to have an absolute dependence on the change of the fundamental bass.—I shall make some observations on fig. 1, which, as there is very little difference between this and fig. 2, may be applied equally to both.

When the fundamental bass moves from 1 to 3, the upper melody rises a semitone major $\frac{16}{15}$; the second melody a tone minor; and the third is a ligature, or continues in the same tension.

The next movement of the fundamental bass is from 3 to 9, which raises the upper part, or melody, a tone minor higher; the second is a ligature; and the third part, or melody, ascends a semitone major.

The third and last movement is from 9 to 3, which obliges the upper melody to ascend a tone major; the second is a ligature; and the third melody descends a semitone major.

Thus we discover three things in this composition of the tetrachord.

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The first is, that when the bass moves from 1 to 3, or from 3 to 9, there are only two sounds that move among the harmonics, and the third sound is a ligature, by being common to both.

Of these two sounds that move, one is either a tone minor $\frac{10}{9}$, or major $\frac{9}{8}$; and the other is constantly a semitone major $\frac{16}{15}$.

Now as the semitone major and the ligature are found to attend every change of the bass, they are without doubt a distinguishing part of the character of the diatonic gender, and will, as such, be constantly found among the harmonics of the fundamental basses in the ratio of 3.

In each tetrachord the semitone is marked with a horizontal line, and the ligature with a semi-circle or bind.

The next observation to be made is, that no tone major $\frac{9}{8}$ is found among the melodies, till the fundamental bass moves from 9 to 3, and then that interval appears, and completes the formation of the tetrachord in the upper melody; for as there are but three species of intervals in the diatonic system, it seems necessary for the principal melody to be composed of them all, in order to establish a system, founded in nature, on the most certain and obvious principles.

The third observation is, that the tetrachord is found to be a particular melody, generated by the fundamental bass, and subject to harmonic powers, and must, therefore, be considered

sidered in a secondary relation with respect to first principles, directly contrary to the method observed by all Theorists, who have made it a first and original one, by mistaking the effects for the cause, without any regard to minor or major scales: by which means they have confined their observations to a simple melody, contracting a system founded on harmonic principles to three simple intervals, without considering that the scale or system of the diatonic octave was so blended and united with the tetrachord, that all its intervals were determined by the same three terms, which formed the harmonious constitution of the tetrachord.

To illustrate this truth, I have at fig. 3 and 4, Pl. XXII, selected from among the harmonics of 1, 3, 9, of the tetrachords, fig. 1 and 2, the sounds proper for a scale of diatonic degrees, which are ranged and disposed agreeably to the order of minor and major scales, together with the roots or fundamental basses, to which they respectively belong. But as they stand opposite to each other, we more readily discover the difference between intervals formed for melody in the diatonic, and those proper for a bass; or, as the Grecians have distinguished, between systematic and diastematic intervals.

To understand the composition of these scales, it must be observed, that the roots, 1, 3, 9, are placed in the first or lowest space; the fundamental basses are in the second, expressed by the multiples of the roots, the more easily to discover the interval formed between the system or scale, and the bass, which is set down as a ratio in the third space. The fourth space contains the numbers, which are to be compared
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with those of the fundamental basses; and the fifth space contains the letters which those numbers represent, together with the ratio of the interval, from any letter to the next.

This ratio is discovered by comparing one number with the next in the fourth space; thus, $\frac{3^6}{4^2} = \frac{9}{10}$, and the like. The ratio, or interval between the bass and the melody, is discovered by comparing the numbers in the second space, with those of the fourth; thus, $\frac{3^5}{7^2} = \frac{1}{2}$ an octave.

I have been particular in explaining these scales, because by the same method the tetrachord and its harmonics, and the scale which is included in that system, may be formed from any three next terms in the harmonic tables, as 3, 9, 27, or 9, 27, 81: observing, however, that in order to bring the numbers nearest to each other, we may multiply or divide by 2 any number in the tetrachord for that purpose; thus, in the present case, in the tetrachord fig. 1, C 30 is an octave different from C in the scale fig. 3, because 60, 64, is nearer each other than 30, 64: also, at fig. 2, E 60, is not so near as 30 is to 32; the first being a compound, and the last a simple ratio.

I shall now examine the system of diapason or octave, as it has had a preference to the tetrachord by some modern theorists.

In the first scale, fig. 3, Pl. XXII, the fundamental bass moves from E 48, to D 54; or, which is the same, from 1 to 9. Also in the second scale, fig. 4, from F 32, to G 36; or,

or, as before, from 1 to 9, which in its nearest degree is a tone major, $\frac{8}{9}$.

Thus the original character of the fundamental bass, which had its motion from a geometric progression in the ratio of 3, is lost; and the diatonic is substituted in its place as a bass, contrary to every principle of harmonics.

From these observations we discover the impossibility of forming a theory of harmonics from the system of diapason; for it appears, that this series is not capable of continuing a succession from one extreme to the other, as there is no communication in either scale between 40 and 45; or, which is the same, between 1 and 9, and the utmost that can be made of the system of diapason has already been explained in page 122. (Rule XI.) In which an incomposite interval preserves the geometric progression of the bass, by the interposition of 48, between 40 and 45; after which, the remaining part of the series is perfectly regular, and conformable to the three terms in the ratio of 3.

Thus the perfection of the tetrachord, and the reputation of those, whose theories for many ages have been regulated by its harmonic powers, must be acknowledged; but as this is not the only instance in which it is found superior to the harmonic system of diapason, or to any other system, I shall proceed to the conjunction and disjunction of the tetrachords.

SECTION III.

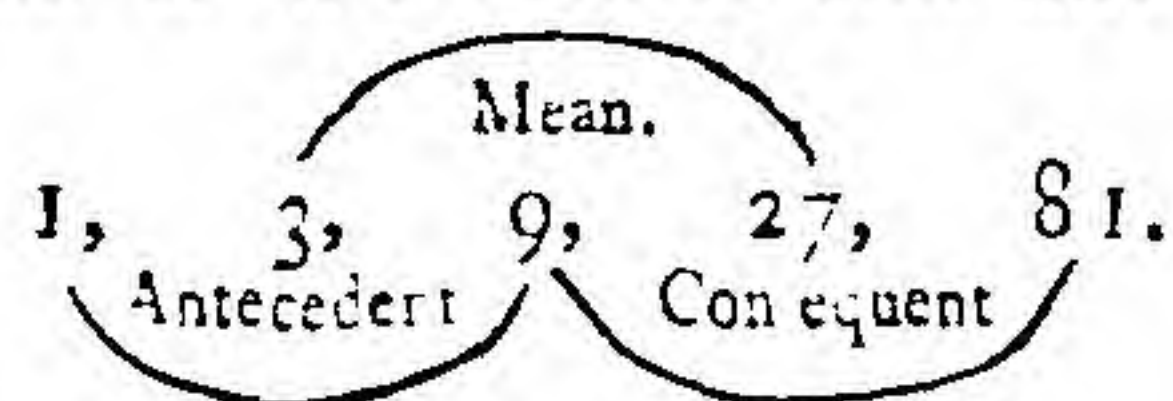
OF THE CONJUNCTION AND DISJUNCTION.
OF TETRACHORDS.

This distinction of *Conjunction* and *Disjunction* will best be understood by considering, that as a tetrachord is the product of any three next terms in the geometric progression, it follows that any new term added must unavoidably make some alteration among the harmonics; in consequence of which, a new scale will be formed. For this reason, it is absolutely necessary to fix and determine these alterations by principles the most certain.

For this purpose I shall take three tetrachords, and place them as antecedent, mean, and consequent, in order to compare the harmonics of one tetrachord with those of another, to discover their difference.

Thus they will stand as at Pl. xxiii and xxiv, and the

progression will be



I shall begin with the mean, 3, 9, 27, and comparé it with the consequent 9, 27, 81, to which it is conjoined: for it must be observed, that the mean has no other motion, than to the antecedent, or consequent.

As it is indifferent which diagram is taken for this comparison, the ratios being the same in both, I shall take those at Pl. xxiv of the second diagram, fig. 2.

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In comparing C, root 3, with the major 3d, C #, root 81, a semitone minor of the greatest magnitude is found in the ratio of $\frac{135}{128}$, instead of $\frac{25}{24}$. Also the 3d to root 3, is a comma different from the 5th of root 81. (See the harmonic tables.) As the same irrational intervals exist in comparing the mean tetrachord with the antecedent, they need not be repeated; but we may proceed to the antecedent and consequent. In comparing a multiple 256 of root 1 F, with 243 E, which is the fifth to root 81, we have the limma of the ancients; also in comparing 32, a multiple of root 1, with root 27, we have a 3d deficient by a comma $\frac{32}{27}$.

Besides these defective intervals, others may be found, if these were not sufficient, to shew the impossibility of their existing in one and the same scale; and therefore they cannot affect the composition of the tetrachord, which has been found extensive in its harmonic powers, and perfect in all its parts. Instead, therefore, of objecting to these intervals, which are only to be found in comparing the composition of one perfect system with another, they are so many proofs of the perfection and regularity of this theory, in which no new term can be added to the first three, without giving immediate information of the change of the scale, by the alteration of one, or more ratios; for as long as the geometrical progression is observed in the roots, so long each scale will be perfect: and if we consider the various revolutions among the harmonics, which must unavoidably happen between 80 and 81; or, 128 and 135; or, between 243 and 256; it will be impossible to discover any interval among any three next terms, that is not the most perfect, as well as necessary for the mu-

tations of the scales; for as the succession of sounds is natural and agreeable, the variety which they produce by the mutations is one of the most animating parts of musical composition.

Thus the origin and necessity of these intervals are discovered, and the objections of modern theorists removed.

The *Disjunction* of the tetrachords has the same principles for its foundation, as the conjunction; viz. the motion of the fundamental bass, in a geometric progression; to preserve which, this distinction is unavoidable in a theory of harmonics, but does not appear of any consequence, when considered only as a melody, independent of the fundamental bass.

In Pl. xxiv, there are three tetrachords and two disjunctions, fig. 4.

These tetrachords are formed from the same roots as those above, fig. 2, but are disposed in a different order. The first tetrachord begins at 9, and ends at 27; the next begins at root 3: but as there is the same objection to 27, 3, as there was before to 1, 9; or 8, 9; tone major, the interposition of another root, 9, at the disjunction becomes absolutely necessary to preserve the progression in the ratio of 3, as 27, 9, 3, and not 27, 3.

The same observation is to be applied to the last note at 9, in the second tetrachord, and the first of the third, root 1; where, for the same reason, 3 must open the communication between 9 and 1, to preserve the progression 9, 3, 1, and prevent

vent a diatonic motion in the fundamental basses : but, besides this, there are two other reasons of equal importance ; the first is the mutations of the scales, which decrease the number of flat dieses, as at fig. 3, Pl. xxiii, and also the sharps, fig. 4, Pl. xxiv. For from two flat and two sharp dieses, which are necessary among the harmonics in the first tetrachord, fig. 3 and 4, we change to one in the second, and proceed to the natural scale at the third tetrachord ; but the contrary of this may be observed in conjoined tetrachords of each diagram ; where, from the natural scale, the mutation was made first to one, and then to two dieses. See fig. 1 and 2, Pl. xxiii and xxiv. The second reason is, that the melody descends in the second diagram, and ascends in the first, a tone major from the last note of one tetrachord, to the first note of the next tetrachord, by disjunction ; which the Greeks distinguished by the name of the diezeutic tone, or tone of disjunction : now as this is a tone major, and is preceded by a tone major in the former tetrachord, and as two tones major can never succeed to each other in the same scale, the disjunction became absolutely necessary, not only to form a new phrase and a new scale, but also to prevent the disagreeable effect of a very sudden and unexpected change of the dieses, in moving from root 27, to 3.

From the nature of this tone of disjunction we may acquit the Grecians of the charge, so often brought against them, of forming their diatonic gender with tones major and limmas only ; since the harmonic explanation of their tetrachords proves the contrary.

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It must also be observed, that the ligature and semitone, two principal characters among the different melodies of every tetrachord, as has been already explained, are formed by the motion of the fundamental bass in the ratio of 3, and attend every such motion; and that the flat and sharp dieses begin in each diagram at root 27, and may be continued in an increasing series, in proportion to the number of conjoined tetrachords, from one to five, or more, dieses; the flats descending by fifths, and the sharps ascending by fifths, in this order.

$$\left. \begin{array}{l} \{ B \flat, E \flat, A \flat, D \flat, G \flat, \text{ descending.} \\ 1, \quad 2, \quad 3, \quad 4, \quad 5, \\ \{ F \sharp, C \sharp, G \sharp, D \sharp, A \sharp. \text{ ascending.} \end{array} \right\}$$

But contrary to this, the disjunction of the tetrachords decreases the number of flat or sharp dieses, from 5 to 4, and 3, &c. and thus the mutations may be made from any given or principal scale a fifth higher, or a fifth lower, by the increase or decrease of the flat, or sharp diesis; or, which is the same, by the conjunction or disjunction of the tetrachords.

From these observations on the conjunction and disjunction of the tetrachords, the distinction between them appears the most necessary to support a theory in all its parts, on the most satisfactory and rational principles. We have seen several melodies determined by the progression of the fundamental bass, from which different scales have been formed, and the number of dieses peculiar to each has been limited, without a possibility of the least alteration, unless a new term is introduced, which leads either to a conjunction, or a disjunction of the tetrachords,

in

in which the agreement between the scales is found the most perfect; and the irrational intervals, which have already been explained, are removed to too great a distance to give the least offence. Thus in the present case, if E, D, is a tone minor in the antecedent tetrachord, fig. 2, Pl. xxiv, and a tone major in the consequent, yet, as we must pass through the mean or middle tetrachord, before such alteration can take place, no disagreeable consequence can be discovered, especially as the mutations are supported by the progression of the fundamental bass, in the ratio of 3, whose harmonics are always so perfect, as to admit of no alteration whatever, unless a new term be introduced; the consequence of which is, a new scale will immediately be formed in the place of the former, without any imperfection in the ratio of its harmonics.

SECTION IV.

EXPLANATION OF THE QUINTUPLE PROPORTION.

The three *primes* are 2, 3, and 5; but as a progression of the first, 2, 4, 8, &c. can yield no variety, the harmonics being the same in all; and as the triple progression 1, 3, 9, &c. has been fully explained, there remains only the quintuple to be considered, by which we shall discover a *new* motion in the fundamental bass of ascending, or descending a third major or minor, which by a judicious mixture with the triple proportion, will produce a variety of intervals not yet investigated, and add greatly to the powers of the mutations.

Among

Among the many discoveries made in this theory by the triple proportion of the fundamental bass, no semitone minor has yet been found in the ratio of 25, 24.

The usual method of discovering this interval has been by dividing the tone minor into two parts; or, which is the same thing, subtracting a semitone major from a tone minor, $\frac{10}{9} - \frac{16}{15} = \frac{25}{24}$, and the remainder is a semitone minor. (See Rule 15, p. 126.) If this interval were the only object in view, we might be content with either of these methods.

Some have asserted, that the ancients had not 5 among the number of their primes, yet at the same time acknowledge that the enharmonic diesis is an interval found in their theories, which is in the ratio of 128, 125. Now as 125 is a multiple of 5, it follows that 5 must be one of the primes, as well as 2 and 3. The arithmetic series, which has all along been applied to the roots in the triple progression, will in like manner be applied to the quintuple, and will demonstrate this truth.

The quintuple proportion is the only one now left to complete a theory of harmonics; for, besides the semitone minor, the fundamental bass will acquire a new motion of ascending and descending a third, and a very considerable addition will be made to each of the diagrams; for the first, which consists of scales with minor thirds, will be improved with those of major, and the second diagram, which consists of scales with
major

major thirds, will in like manner have their respective minor scales.

The quintuple progression, like that of the triple, is of no consequence in harmonics unless united with the arithmetic series: for from the first to the third term in each is discord, as $\overbrace{1, 9}$, or $\overbrace{5, 125}$; and therefore each as a root, must be multiplied by the arithmetic series 4, 5, 6, or 6, 5, 4, according to the diagram to which they are to be applied. Thus $5 \times 6 = 30$, $5 \times 5 = 25$, $5 \times 4 = 20$, and the products are the harmonics to that root, 5; also $25 \times 6 = 150$, $25 \times 5 = 125$, $25 \times 4 = 100$, are the products, or harmonics of root 25; and if the same multiplication is repeated at root 125, we shall have the harmonics to that root, or to any other in the quintuple; but it must be observed that 1, 5, or 5, 25, are double compounded ratios, being each a seventeenth major; yet as F, A, or A, C#, are major thirds without any distinction of compound or simple thirds, so these are brought the nearest into a simple ratio, 4, 5, or 20, 25, which is the same; in like manner as 1, 3, was before brought to 2, 3; but this will not be difficult, as the letters are annexed to each number in the harmonic tables of the quintuple, which immediately succeed those of the triples in each diagram, Pl. xv and xvi, which have been explained page 115. But as the roots in these tables move in the ratio of 3, as 5, 15, 45, there are two other Plates, xxv and xxvi, which are intended to discover the easy communication between the minor and major scales, with the same number of flat or sharp dieses in each diagram; by which the mutations from a minor to a major scale, or the contrary, will be readily understood.

X

Each

Each of these tables is divided into four sections. The three columns of the first section contain seven roots from unity to root 729, in the ratio of 3, together with the letters, and the numbers which express their relation as harmonics to each root, in the same manner as they stand in the other harmonic tables.

In the second diagram the 3d to the root in the first section is the root to the second throughout, after being brought to its lowest denomination. Thus, 20 is reduced to 5, the root of the second section, which, when multiplied by the arithmetic series 4, 5, 6, or 6, 5, 4, gives the harmonics of root 5, as 20, 25, 30. Also if 60 be taken as a root, its lowest form is 15, which, being multiplied as before, gives 60, 75, 90, for its harmonics.

It must be remembered, that in the first diagram of the quintuple, (Pl. xxv.) the upper letter is the root, and the lowest letter in each root is the fundamental through every section.

As there is no alteration in the thirds to each fundamental, the fifth to each root in the first section is applied as a third to that of the second section; as from 24, root 1, to 20, root 5; by which means we have two species of thirds to each root, or fundamental, throughout the three last sections. Thus, 24, from root 1, is applied to root 5, and 72 to root 60, or 15; in consequence of which, we have the two species of thirds to each fundamental, and the semitone minor in its true ratio 24, 25; and by pursuing the same method with the other five roots the same advantages will follow.

The

The third section is formed from the second, and the fourth from the third, in the same manner as the second was from the first; that is, the major third to the root of the second section is 25, which is the root of the third; and the third major to the root of the third section is in like manner carried forward, as a root to the fourth section 125, and the other operations follow the same as at first; and each of these roots is multiplied by 4, 5, 6, or 6, 5, 4.

Every three next terms in the same section, by being in the ratio of 3, form a tetrachord, and consequently a scale peculiar to those three terms only; and the mean or middle term is the principal sound, or tone, to which all the other sounds in the scale refer, as the generator of the whole; in the same manner as has before been observed in the triple progression. There is, however, a very great difference between these two proportions: for any three next terms in the triple may move in progression, as 1, 3, 9; but it is not so in the quintuple, 1, 5, 25, or 5, 25, 125.

Therefore, as the triple proportion has been found to be the original of the quintuple, and that the latter has not, like the former, a progression of three terms, it follows that the motion of the fundamental bass in the quintuple must return to that of the triple, immediately after the change of rising or falling a third.

Thus in the first diagram, Pl. xxv, the motion from the fundamental bass A 72, to F 90 and B \flat 270; or from A 72, to F 90 and C 30; is better than 72, 90, and D \flat 450; or from D \sharp 216,

X 2

to

to F 90 and A \flat 150; because the fundamental bass, by moving first a third and then one or more fifths, in the triple proportion, leaves us in no kind of suspense as to the mutation of the scale; but a motion of two greater, or two less, thirds in succession throws all the obscurity that is possible on the mutation, and leaves us in a painful uncertainty, till it is succeeded by the triple proportion to determine the scale.

The same observations may be made on the second diagram, Pl. xxvi, where the motion of the fundamental bass from C \sharp 48, to E 60 and A 20, or from 48, to 60 and B 180, is very good, and leaves us in no doubt with respect to the scale; but it is not so from 48, to 60 and G \sharp 300, in which the motion of two major thirds in succession in the bass makes so great an alteration in the ratios of the intervals, as cannot be admitted, unless on some very extraordinary occasion, to support a particular sentiment, in which reason has little to-do in the government of the passions.

The motion from F 16 to C \sharp 100; or, which is the same, from F, root 1, to C \sharp 25, or from A, root 5, to E \sharp , root 125, is equally exceptionable with the former; because in this case the fundamental bass moves an interval, consisting of two major thirds, in the ratio of $\frac{25}{15}$.

As it seems contrary to the quintuple proportion to move a fundamental bass a minor third, it is therefore necessary before we proceed to explain the nature of this motion.

The

The change of harmonics in the triple proportion, as from 1 to 3, or from 3 to 9, has been always attended with a ligature, or one sound common to both roots; agreeably to the rule which Euclid (p. 21) has laid down to be observed in the mutations.

This will appear at Pl. xxv, in which the fundamental bass of root 9, which is D 216, may move a minor third to F 90, which, in the quintuple, is the product of the bass A 72 at root 3; for in this change we have one, or, if we please, two sounds that are common. But if A 72, at root 3, should move to B \flat 270, the product of root 9, we have no ligature or common sound attending the change; and therefore it appears that there is no communication between 72 and 270, without first moving to F 90, the product of 72.

Hence we have a positive law, which forbids a fundamental bass to move a semitone, in like manner as before to move a tone major 9, 8, in the triple progression.

The same rules must be observed in the second diagram, Pl. xxvi; where G 144 may move to E 60; but C 48 cannot move to B 180, for the reason before given, not having a common sound.

Thus it appears that the motion of a minor third in the fundamental bass has its original in the quintuple proportion, as well as the major third; and the laws which govern one, are equally applicable to the other.

These

These and the preceeding tables of the first and second diagram contain all the various species of intervals, both rational and irrational ; for by uniting the arithmetic with the triple proportion, and sometimes with the quintuple, the scale or tone is perfected, as well as the mutations proper for every mode.

We have also not only two scales with minor and major thirds, peculiar to one or more flat or sharp dieses, but the two natural scales with different species of thirds.

Each scale is the product of any three next terms ; and the second, or mean term, is the tone or principal sound of that scale. The relative scales, connected with the principal one, are regulated by the increase or decrease of one diesis more or less than what completed the first proposed scale ; whereby three minor and three major scales may be used in every mode. But these five auxiliary scales make no alteration in the name of the mode, which is notwithstanding distinguished by that of the principal tone of the scale first taken. Thus if we take any three next terms, as G 9, D 27, A 81, which may represent as many principal tones of as many different scales, the second or mean term 27 is that which governs the whole, and gives its name to the mode. And agreeably to what was observed before of the increase or decrease of the numbers of dieses, 27 has two, 81 has three, and 9 has only one diesis ; and as the formation of two scales of a different kind depends on the same number of dieses, we have six scales in the whole, which complete the mode and all its mutations

But

But as an example on this subject will better be understood, I have, at Pl. xxvii, extracted from the first diagram, Pl. xxv, three roots proper to form three scales, as at fig. 1, in which the principal tone of each is in the proportion of 9, 27, or 81, with a minor third to each; and if considered as a mode, the scale which governs it is the second or mean term 27, which, to be perfect, requires two dieses, with three at root 81, and one only at root 9.

Opposite to these, at fig. 2, are the roots of three more scales, in the same proportion of 9, 27, 81, notwithstanding they are expressed by large numbers.

Each of these has major thirds, and the same number of dieses with those which are parallel with them, to form their different scales. Thus G 27, a minor third, with two dieses, and B ♭ 270, with two dieses a major third, correspond in like manner with each other, as does D 9 with F 90, or C 81 with E ♭ 810.

The pitch of the principal tone of the mode may be at G 27, with a minor third, or at B ♭ 270, with a major third, and the other five scales are the auxiliaries; and the whole being regulated by the number of dieses, forms one mode only, according to the doctrine of the ancients.

From this explanation of the geometric proportion in the ratio of 3, it appears to be an *universal leading principle* in a theory of harmonics, and extends its powers through every essential part.

Thus

Thus the three terms of every scale or tetrachord, the three first tetrachords of the Grecian diagram conjoined, or, which is the same thing, the three principal scales in the same kind of every mode, are formed by these proportions. Besides which, the three hexachords of Guido, the moll, the natural, and the quadro, together with the three cliffs of the moderns, F, C, and G, the order also and succession of the flat and sharp dieses are governed and regulated by the same geometric proportion in the ratio of 3; and as the two Grecian diagrams have already been found throughout in the same ratio, there can be no doubt of the consummate knowledge which the Greeks had of Music as a science, and of Harmony; without which a progression in the ratio of 3 could be of little or no use.

SECTION V.

OF THE MUTATIONS.

The Mutations (before fully explained according to the directions given by Euclid) may be reduced to the five following rules:

I. A mutation, or change from one scale to another, may be a fifth higher or lower; that is, from 27 to 81, or the contrary; or from 27 to 9, and the contrary. See Pl. xxvii, fig. 1 and 3.

II. A third higher or lower, which will be from the minor to the major scale, or the contrary.

III. A

III. A dissonant mutation may be by rising or falling a tone 9, 8, which is done by moving from 9 to 81, or the contrary.

IV. A mutation is made by changing the genus of a scale; as from D with one flat to D with two sharps, or the contrary.

V. By a change of the mode; when the first movement is in one mode, and a second movement succeeds in another, which sometimes bears a relation to the first, and at other-times none at all.—Instances of this are to be met with in many Cantatas, Recitatives, &c.

SECTION VI.

OF DISCORDS.

The motion of the fundamental bass having been determined by the triple and quintuple proportion, and their several concords discovered by the arithmetic series, it remains to consider the origin of discords, their different species, and how they become an essential part of harmonics.

The ancient writers, in treating of discords, have passed rather too slightly over a subject of so great importance.

Aristoxenus, in his division of intervals, says, (p. 16) “The first division of intervals is that by which they differ from among themselves in magnitude.”

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“ The

“ The second, that by which the consonant differ from
 “ the dissonant ;” (p. 45) and yet he gives no other rule of
 dissonance, or discord, than the difference between diapente and
 diatessaron, which is a tone major 9, 8.

Euclid, in his division of intervals, (which I have before
 taken notice of) says, (p. 8) that “ The consonant intervals
 “ differ from the dissonant ” and that “ the latter are placed
 “ between the former ;” and then proceeds to tell us, that
 “ consonance is a mixture of sounds differing in acuteness and
 “ gravity, but that dissonance, on the contrary, refusing to mix,
 “ hurts the ear with harshness.”

From a writer of Euclid's eminence more might have been
 expected, especially as Aristoxenus had said so little on the
 subject.

Nicomachus comes nearer the point ; for he tells us (p. 11,
 &c.) that Pythagoras, in passing by a smith's shop, heard dif-
 ferent sounds, from the striking of hammers on an anvil, which
 engaged his attention so much as to put him upon trying some
 experiments, to discover the exact relation those sounds had to
 each other : and after different trials of the weights, he pro-
 ceeded to try some experiments with strings, and discovered
 the proportions of 6, 8, 9, 12 ; and by comparing either of
 the extreme with either of the mean proportions, as 6, 8, =
 $\frac{3}{4}$, or 6, 9, = $\frac{2}{3}$, or 12, 9, = $\frac{4}{3}$, and 12, 8, = $\frac{3}{2}$, he discover-
 ed the diatessaron and diapente in their true ratios, and also the dia-
 pason from the two extremes, 6, 12, = $\frac{1}{2}$: but with respect to
 the

the mean proportions 9, 8, he found no agreement in their relation, and therefore pronounced them discord; and by inserting the several degrees of sound between 6, 8, 9, 12, he discovered the diatonic gender, composed of two tetrachords disjoined, equal to the system of diapason.

What use Pythagoras made of this discovery we are not informed, for Nicomachus, who wrote this little manuel in a hurry to oblige a lady, excuses himself from being more particular by the shortness of the time, but promises to treat every part more at large in his Commentaries; however as no such book has come down to us, we must be satisfied with what is now before us.

This account of Pythagoras has been objected to by many writers; but as the numbers and their ratio correspond with the different lengths of strings and their vibrations, it is sufficiently satisfactory on the present occasion.

In examining these proportions 6, 8, 9, 12, we find,

1. That by reducing these numbers to their lowest denomination we discover the three roots or fundamental basses, 1, 3, 9. See the two first columns in Pl. XXVIII and XXIX.

2. That by applying the arithmetic series to each of these three terms, the system of the tetrachord and the scale have already been formed in each diagram. Pl. XXII.

3. That the character of a principal sound or generator has been discovered in the mean or middle term, by the application

of the two first original principles of the different lengths and their vibrations. See Pl. XII; in which, the fifth above and fifth below unity are in the ratio of 3, which answers to the series, 1, 3, 9.

4. The fifth, being thus generated by either of these principles, discovers its union, as a principal harmonic sound with every fundamental bass, whether it has a consonant or a dissonant combination applied to it.

5. It has before been observed that the two products of 3, a generator, 1, 9, or 9, 1, cannot *succeed* to each other as fundamental basses; but this does not prevent their being *united* in harmonic combination; especially if by this method the different species of discords are formed, agreeably to the observations made by Aristoxenus, Euclid, and Pythagoras.

The first says, that "a discord is the difference between diatessaron and diapente;" which corresponds with the 9, 8, of Pythagoras; and Euclid says, "that the discord is placed between the concords;" in like manner 9, 8, is placed by Pythagoras between the concords, 6, 8, 9, 12. But Euclid proceeds further, and says "the discords will not mix like the concords;" Pythagoras removes this objection, by *combining* the discord with other sounds that will mix; of which the fifth, as was observed before, must always be one, as appears in this Pythagorean combination, 6, 8, 9, 12.

That this and other dissonant combinations, discovered by the union of the first and third terms, 1, 9, or 8, 9, may be better understood,

understood, I have placed them in the two Pl. xxviii and xxix.

The first is agreeable to the different lengths of strings, the second to their vibrations; and as the species of discords are the same in both, so the observations, which I shall make on one, may be applied to the other, except the difference which is found between the minor and major third; which will be taken notice of in its proper place.

The first column in Pl. xxix has the three terms, or fundamental basses, 1, 3, 9.

The second has the *Pythagorean* numbers 6, 8, 9, 12.

The third has the harmonics to the three terms.

The fourth column has a discord, which is the suspension of the fourth formed on C, a fundamental bass, to which F is a fourth, and is a discord to G, a fifth to the root C.

In the fifth column is another discord, viz. the suspension of the ninth, formed on F, a fundamental bass, to which A is a third major, C a fifth, G a ninth and a discord to the root.

The sixth column is a third term compounded, having a minor seventh to G, a fundamental bass, to which B is a major third, D a fifth, and F the discord is a minor seventh, and forms the compounded harmonics of the third term.

The

The seventh column is a double compound, having a ninth major or minor to G, a fundamental bass, to which B is a major third, D a fifth, F a seventh, and A \sharp , or \flat a discord and a double compounded third term, as in the first diagram the seventh column is a double compounded fourth term. Thus, by the union of the first and third terms, 1, 9, or 8, 9, these three species of discords are formed, and supported by their different fundamental basses, and their fifths.

The additional harmonic powers, which these three terms, or fundamental basses, acquire by the application of the discords, contribute very much to distinguish their different characters, by which their succession is easily discovered; as for example;

The first term from its situation, is incapable of receiving any other discord than the suspension of the ninth.

The second term may have either of the suspensions, but not the minor seventh.

The third term has more extensive powers than the other two; for besides the suspension of the fourth, or the ninth, it can receive the minor seventh and major third, which is peculiar to that term only, as will be explained in page 168.

Hence we may observe, that when Pythagoras determined 8, 9, to be discord, he could not, as many have imagined, intend a mere succession of sounds; (for then the tetrachord and other perfect systems must have been a succession of discords;) but on the contrary, a combination with other sounds. For if we consider, that all communication is cut off between
the

the first and third term, 1, 9, as fundamental basses, it seems as if their *union* was reserved to be exerted in a fresh creation of harmonics, and to give new and distinct characters to the three terms.

Having explained the discords in Pl. xxix, I shall now compare them with those in Pl. xxviii.

The first five columns are the same, except that minor thirds are next the fundamental bass; but the sixth column in each being compared discovers a difference of very great consequence to them of the fundamental bass.

Each contains a seventh and a discord; the first has been found to have a major third, but the other on the contrary has a minor third next the fundamental. These discords, when compared with their respective fundamental basses, are found in the same ratio; but when compared with the other harmonic parts are very different. Thus,

Pl. xxix, from E to D, is a minor 7th in the ratio of 16, 9.

Pl. xxviii, from G to F, is also a minor 7th in the ratio of 9, 16.

Pl. xxix, from G to D, is a 5th deficient by a comma = 40, 27.

Pl. xxviii, B to F, is a semidiapente or imperfect 5, = 45, 64.

Pl. xxix, from B to D, is a minor 3d deficient by a comma, = 32, 27.

Pl. xxviii, from D to F, is also a minor 3d deficient by a comma, = 27, 32.

In

In comparing the seventh with the basses and their harmonics, we discover no other difference than that which arises from the two species of thirds to the fundamental basses; for these thirds, when compared with the sevenths, give two species of fifths, which is confirmed by adding each species of third to its particular fifth, and the product is the ratio of the seventh;

$$\frac{5}{6} + \frac{2}{3} = \frac{9}{16}; \text{ also } \frac{5}{4} + \frac{6}{45} = \frac{16}{9}.$$

However imperfect these fifths, as well as the deficient thirds may appear at first sight, yet as they are occasioned by the addition of a new harmonic, to which the others are compared, they cannot be objected to, not only because the whole combination is extracted from the scale, which admits of no sound that is not a product of the three terms, but also because each interval in the whole combination, when compared with the fundamental basses, is in its true ratio.

Hence it appears that all harmonics, whether simple or compound, are not to be questioned as to their mutual relations, provided they are true to the fundamental bass, whose powers they are intended to encrease, by supporting its operations.

This will appear very evident in the distinct characters of these two sevenths, which I shall explain in their turn: but as that with a major 3d, Pl. xxix, has something very interesting in its composition, I shall begin with that first.

This seventh is composed of three species of thirds, from G to B, B to D, and D to F, in the following ratios;

$$\left(\frac{5}{4} \right) + \left(\frac{6}{5} \right) + \left(\frac{52}{27} \right) = \frac{16}{9}.$$

The

The D, and the F, which make the deficient third by a comma, are not only original sounds in the scale, but are also found in the geometric progression, 1, 3, 9, 27, and therefore are fixed and unalterable, whether they appear as fundamental basses, or harmonics.

The active principle of the semitone major has already been observed in the formation of the tetrachord, Pl. VII, Fig. 2 and 4, which is constantly formed, when the bass moves in the ratio of 3; and as there are two semitones in every scale, one of which attends the bass, from 1 to 3, and from 3 to 9, or the contrary; so, in this compounded harmonic, now under consideration, these two semitones move in a contrary direction, when the bass moves from 9 to 3. Thus the major third to 9 ascends a semitone major to the octave 3, the second term; and the discord, or minor seventh, descends a semitone major into the third, to root 3, the second term.

From the motion of these two semitones, we may observe two things of equal importance:

1. That all original discords must be considered as minor sounds, and must descend to the next degree to be resolved.

2. That as the major third ascended, so in like manner all major sounds must ascend to the next degree, and cannot descend without changing the scale by introducing a new term.

Z

From

From this natural motion in the minor and major sounds, we are conducted with the greatest certainty to the second term, or principal generator; as if by its powerful attraction it drew, and resolved every sound to itself, in order to finish the phrase with the most perfect termination.

From the vast variety found in harmonics, we may have as many basses as there are notes or letters in the combination. Thus this minor seventh, by changing the position from G, B, D, F, to B, D, F, G, or D, F, G, B, or to F, G, B, and D, forms other harmonics; but the first only is fundamental; and the other discords, which proceed from these inversions, are formed by the different positions of the relative basses, and distinguished by particular figures placed over each, to shew their relation to the fundamental bass.

These different basses diversify the proportions; and by placing the same thing in a variety of forms, the first impression is changed to others no less characteristic and entertaining. Thus G, the fundamental, may rise a major third to B with a sixth and imperfect fifth, $\frac{6}{5}b$; or to D, a fifth above, with a compound sixth, $\bar{6}$, or, to F, a seventh above with a major fourth, or tritonus, and a second, $\frac{4+}{2}$.

These transpositions of the original combination do not alter the nature of the discord at F, or the major sound at B, since in every position the F must descend to the next degree to be resolved, and the B must ascend a semitone at the instant

instant the bass moves from 9, the third term, to 3, the second.

Hence we discover the difference between an original discord and others which are accidental; for if we take away the minor seventh F, we remove at the same time all the accidental discords, viz. the semidiapente or imperfect fifth, the fourth or tritonus, the second, and the deficient third, 32, 27; and the simple harmonics only remain, as at first, in their most perfect form.

I must observe that as melody depends so much on a diatonic succession of sounds, which have been discovered to be the immediate products of the fundamental basses, it appears here, that by the assistance of this minor seventh, we have a system formed by a communication from the octave of the third term to its seventh or minor sound, which proceeds to the third of the second term, as thus, G, F, E.

This succession is not only very agreeable in itself, but gives a character to the seventh, which cannot be allowed to either of the suspensions, the ninth or the fourth; that is, this seventh may be used without being prepared, though, like the other two discords, it must be resolved by descending to the next degree.

I have thus explained the most remarkable parts of these combinations, by which the character of an original discord is distinguished from those that are accidental; together with the

active principles of the two semitones, which direct the motions of the minor and major sounds, so as to determine the motions of the original, as well as the accidental, discords in their different positions; by which we are conducted to the second term, or principal generator, and its harmonics, which are the several points to which these motions are directed, as the final or perfect repose.

Thus the same laws, which instruct us in the *formation* of the mass of harmony, teach us also how to give it *life* and *motion*; and if we attend to the several impressions made on our ear, by this dissonant combination, transposed to different positions, we discover that our expectation of the succeeding second term almost anticipates the change, and conducts the ear to its effect.

Nothing can be more pleasing to a curious and intelligent mind, than to discover the mutual agreement between the laws of science, and those dictated by our own feelings; to be able to account for those charming and agreeable ideas raised in the mind by the powers of harmony, and to reconcile them by principles the most rational and natural.

I shall now examine the other seventh, E, G, B, D, in the sixth column, Pl. xxviii, in order to discover by its motion the term that succeeds to it.

The ratios of the several intervals, which form this combination, have already been mentioned, in comparing them with those of the third term, and the only difference appears to arise from the two species of thirds next to the fundamental bass.

The

The seventh, like the other, must descend to be resolved : but the minor third has no motion like the former, but must continue a ligature at the time of the resolution, when the fundamental bass descends a fifth, by which another seventh is formed. This indeed changes the position of the harmonics, but with respect to the fundamental bass, they stand in the same relation as before, and will continue the same so long as the bass descends a fifth, at the time of the resolution, which moves into a minor third.

Now in a progression like this, where nothing is conclusive, and where even the scale is not always known, some essential alteration in the harmonics to make them more decisive, by bringing us nearer to the second term, is absolutely necessary ; and as a major sound is the only one discovered proper to produce this alteration, the last seventh must be resolved into a semitone major ; by which means we have a major third to the succeeding bass a fifth lower, which gives us the third term.

These harmonics of the third term bring us home to the second term, by the next motion of the fundamental bass a fifth lower.

Thus by the motion of this seventh, we not only discover the term that succeeds to it, but also its character, for as it is antecedent to the third term in the same manner as the third is to the second, it must therefore be a fourth term.

From this additional term, we have two sorts of comparisons in the ratio of 3.

The

The first is, as the first term is to the second, so is the second to the third, which terminates in an imperfect repose, with the simple harmonics on the third term.

The other is, as the fourth term is to the third, so is the second, which finishes and completes the final or perfect repose.

The progression of the fourth term may be continued in one scale, or may move into any other agreeably to the nature of the mutations; but must finally be resolved into the third term, which is decisive with respect to the scale, by being constantly succeeded by the second term.

It appears therefore, that the lengthening of the phrase is the consequence of the use of the fourth term, and in proportion to their number will be the length of the phrase: for as no progression can be formed by either of the three terms in the scale, it is absolutely necessary to find that power placed some where, not only for the sake of the phrase, but also to vary the sentiment, by combinations less perfect, and less conclusive, than the others: all of which are found in the fourth term only, by being farther removed from the second term.

As this fourth term, like the former, is a compounded harmonic, it has also four positions. Thus E, G, B, D, a seventh, or G, B, D, E, a sixth and fifth $\frac{6}{5}$, or B, D, E, G, a fourth and third $\frac{4}{3}$, or D, E, G, and B, a second and fourth $\frac{4}{2}$.

In

In all these positions D is the discord, and must descend to the next degree to be resolved; and if it moves a tone, it forms another fourth term, but if it descends a semitone, it forms a third term on a bass or root, a fifth lower.

As the ratios of all compound harmonics are formed from the products of the three first terms differently combined, so those of the fourth term must vary, according to the part of the scale from whence they are taken; for as no other sounds can be admitted into their composition, the formation of the scale is of the greatest importance in a theory of harmonics.

This discord of the fourth term is sometimes used in a descending system, without being prepared, as was before observed of the discord on the third term; but like that it must be resolved.

From the many observations made on the third and fourth terms, it does not appear that their distinct characters are occasioned by the seventh, when considered simply as a discord, but from its being united with a major or minor third.

It is from this union that the same species of discord discovers a much greater and more interesting variety of revolutions in harmonics than either of the suspensions; and as the seventh is so immediately concerned in all these operations, that even the fundamental bass is limited to descend a fifth, and sometimes a third, at the time of its resolution, and as the motion of the harmonics are regulated by the same cause, it follows that the
seventh

seventh, from its extraordinary powers, must be considered as the *first* species of discord.

From this first species we have discovered a new term, which, by the great influence of the second term in every scale, is placed as antecedent to the third term.

This disposition determines it to be a fourth term, dependent in its resolution on the third; in the same manner as the third is on the second term. It appears therefore that, by the different applications of this first species of discord, and by attending to the motion and character of the fundamental bass, the harmonic powers are greatly encreased.

Hence we have the simple and compounded harmonics limited to the different basses to which they belong: and if the same faithful guide continues to govern and direct the future operations, the double compounded harmonics will be discovered, and finish the characters of the third and fourth terms: as appears in the seventh column of Pl. xxviii, and xxix.

In this theory the motions of the fundamental basses are limited to that of ascending or descending a fifth, and sometimes a third; but there is no instance of their moving a tone, or semitone: from whence the true character of the melody is preserved in its natural motion of tones and semitones, whilst that of the fundamental bass demands a more masculine modulation.

Therefore as long as no other harmonics are admitted, but such as depend on the motion, and are the products, of the fundamental

damental basses, so long will these two opposite characters be preserved, namely, the systematic motion for melodies, and the diastematic for the fundamental basses.

S E C T I O N VII.

Of the double compounded THIRD and FOURTH TERMS.

It has been explained before that the formation either of the simple or compounded harmonics is a composition of thirds, in an ascending direction from a given sound or fundamental bass. Thus as the simple harmonics are a combination of two species of thirds, so another third of either species being added to them forms the compound, or discord of the seventh; or, as before explained, the third or fourth term.

If therefore we advance one step further on the same principles, and add another third to the former, we shall discover the utmost limits of harmonic combination in the double compounded third or fourth term. This harmonic consists of a third of either species, according to the term to which it is applied; of a fifth, a seventh, and a ninth, either major or minor, agreeably to the scale, and the relation that the combination has to the second term.

As this combination, from the variety of its proportions, is one of the most dissonant, and at the same time the most proper for the darker shades of musical composition, it cannot be unworthy of a particular consideration; especially as no

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such

such combination has yet appeared in any theory that I have seen, occasioned by an opinion, that the perfection of the diapason is such, as to contain all other intervals within itself, and that whatever exceeds it, is only a repetition, at the distance of an octave, of what existed before, by which all simple intervals are made compound by the addition of their octaves.

This reason, however specious, when applied to a simple melody, or a single succession of sounds, cannot be admitted with respect to combination, which depends on other principles, drawn from the character and motion of the fundamental bass. But that I may not advance any thing new, in so essential a part of harmonics as this is, without supporting it by the most undeniable evidence, I shall for this purpose select from this theory the following rules.

1. That the whole powers of harmonics are conducted by *four terms*.

2. That the three first are primary fundamental basses with perfect fifths, and generate their own harmonics, from which the scale is formed, and all other combinations peculiar to it.

3. That the fourth term has nothing original in its composition, but is formed from the products of the three first terms, and as its communication with the second term is cut off by the third term, it therefore stands as a secondary fundamental bass, of an inferior character to the third term, to which it must descend a fifth.

4. That

4. That the motion of these four terms in one scale is limited to the triple and quintuple proportion; but in no instance admits of any other, either in the ratio of 9, 8, tone major, or 16, 15, semitone major.

5. That the third and fourth terms may have compounded, or double compounded, harmonics; from which the first and second terms are excluded.

6. That the first and third terms only have an immediate communication with the second,

How far these rules will agree with the doctrine of keeping all harmonics in their first and most simple positions within the limits of the diapason, remains now to be examined.

The extreme flat seventh, and imperfect fifth, or semidiapente, with a minor third, form the combination B \flat , D, F, A \flat , (Pl. xxix.) which now comes under consideration, in order to discover its character as a fundamental bass.

The only sound in the whole scale, on which these harmonics can be formed, is the major sound, which is a product of the third term, and an imperfect concord of a third major. Now if this major sound can support the character of a fundamental bass, all harmonic combination will be confined to the limits of the diapason; but if on the contrary it is found to have no one requisite proper for this character, the double

compounded harmonic must be established as the true and original combination.

It is determined by the first rule, that all harmonic combination is formed on four terms ; therefore, if this major sound could be supposed to be one of the first three primary basses, it must have a perfect fifth, according to the second rule ; which we find it has not : and if it is a fourth term, it cannot move to the second term, agreeably to the third and sixth rule ; and if incapable of this motion as a fourth term, its two minor discords cannot be resolved by any other motion. Hence we discover that a *major sound* cannot be a *fundamental bass*, because its motion of a semitone major, 16, 15, is absolutely contrary to the fourth rule, as well as the whole theory, which limits the motions of the fundamental bass, to the triple and quintuple proportion only.

We may conclude from the fifth rule, that this combination is a part of a double compound ; and as it has no other motion in the same scale, but into the second term, it can be derived from no other than a third term double compounded, agreeably to the sixth rule.

Thus we discover that these harmonics, excluding the fundamental bass, exceed the limits of the diapason, by a semitone in a minor scale, and in a major scale by a tone.

From these observations the first idea of a discord is preserved, which had its original form in the union of the first and
third

third terms; to which is now added the harmonic third of the first term, not by any arbitrary will of our own, but by a necessity, which the motion of the fundamental bass obliges us to comply with.

The extreme flat seventh and imperfect fifth being found as parts of a double compound, peculiar to the third term, it may not be improper to give one example of their formation. In Pl. xxv, root 5, C 30, E ♭, G, are the simple harmonics of the third term, to which the first term B ♭, 270, and its minor third, D ♭, 225, are added, which are taken from the quintuple of root 9, the first term in the triple; and the whole forms a double compounded harmonic on the third term: when the bass moves to F 90, the second term, the two minor discords are resolved, and the major sound E rises a semitone, by which the harmonics of the second term are completed.

Also at Pl. xxvi, the quintuple of root 9, B 180, D ♯, F ♯, a third term, is by the addition of A, C, taken from the quintuple of root 1, the first term, formed into a double compounded third term. But as its motion to the second, at the quintuple of root 3, together with the resolution of its two discords, and the ascending of the major sound, are the same as above, they need not be repeated.

From the same example taken from each of the diagrams we discover how the combination of harmonic sounds and their several relations depend on the fundamental bass; that by its animating powers they receive life and motion; that the character of a third term double compounded is established, in
preference

preference to a major sound, which has no other motion than a semitone; which, for this and other reasons before given, can never be a fundamental bass.

There are two instances in which this combination deserves our particular notice.

The first is with respect to the motion of the semitone major. The active principle of this interval was first discovered in the harmonics of 1 to 3, or from 3 to 9, and the contrary; in which one semitone was formed by the change of each of these harmonics; but when the third term was composed of its minor seventh, we had two semitones formed by its motion into the second term; the one from a minor sound descending, and the other from a major sound ascending, as has been observed before. But in the present case, of a double compounded third term into the second, we have three semitones formed; the flat ninth and minor seventh descend a semitone as minor sounds to be resolved, and the major sound ascends another semitone into the second term, or its octave, which is the same.

The second instance relates to the systems, which make so great a part of the ancient harmonica.

These systems are of different magnitudes; but the least must constitute a minor or major third: which agrees with Euclid's definition, that "the least system contains more intervals than one," (p. 1.)

In moving therefore from the second to the third term twice, with their simple harmonics, we have a system of diatessaron, or
four

four sounds in their next degrees in the melody, as E, D, C, B, beginning with the third of the second term, and ending with the third of the third term, second diagram.

System	E, D, C, B.	}
Roots	C, G, C, G.	}

If the third term is compounded with its minor seventh, we have a system of six degrees, from the eighth of the third term to the third of the same term descending, while the third term moves to the second term at the resolution of the seventh, and repeats the same motion, till the whole is finished.

System	G, F, E, D, C, B.	}
Roots	G, G, C, G, C, G.	}

But when the third term is double compounded, we have a system of seven degrees, while the third term repeats its motion to the second term, during which the system descends from the minor ninth of the third term to its third, which determines the largest harmonic system which can be formed on any two fundamental basses.

System	A \flat , G, F, E \flat , D, C, B.	}
Roots	G, G, G, C, G, C, G.	}

These two last remarks shew us the gradual increase of the harmonic powers, from the simple to the double compounded third term, by the modulation of the semitones, and the variety and extent of the systems.

The

The observations made in the minor ninth, added to the third term, will in all respects be the same if the ninth is major; except, that in the resolution of the discords of the last two semitones only will be modulated, instead of three, which are found in the former; and that the first is formed from a scale with a minor third, the latter from one with a major; but their relation to the second term is the same, because each has a major third and perfect fifth to the fundamental bass, without which, it can never stand as a third term.

Hence the relation that any combination has to the second term must determine the character of the fundamental bass; for if there is no immediate communication with the second term, without first moving to the third, we can be in no doubt of its being a fourth term.

Thus if a double compounded harmonic has a minor third, it forms a double compounded fourth term, and as such leads us by the natural motion of its harmonics to a third term, agreeably to what has been observed before; viz. that the impression made on the mind by a fourth term, whether singly or doubly compounded, is so undetermined, that we wait with a kind of impatience, till relieved by the succeeding third term, which discovers at once the scale, the phrase, the second term, and whatever else is necessary for our information.

The particular composition of this double compounded harmonic, or fourth term, is formed on the same principles with the third term; with this difference only, that if the seventh is minor, the third must be minor, and not major; but if the seventh

venth is major, (which can never happen to the third term,) the third must also be major: as to the ninth, it may be a major, or a minor, third above the seventh, according to the scale.

The two discords, the ninth and seventh, are frequently used on the fourth, or third term, without being prepared, but nevertheless must be resolved: by descending to their next degrees, the root or fundamental bass in double compounded harmonics does not always appear, unless in compositions of four, five, or six parts; but its place is easily discovered by the motion of the harmonics.

SECTION VIII.

THE SECOND SPECIES OF DISCORD.

The principles of the third and fourth terms being established, and supported by the motion of the fundamental bass, the ninth, or *second species of discord*, comes next to be discovered.

To understand its genius and character it will be proper to explain two laws, to which this discord, as a suspension, is subject.

1. It must be prepared.
2. It must be resolved.

By the preparing of a discord is to be understood, that the sound exists as a concord one quantity of time, after which, by the motion of the fundamental bass, the same sound becomes a discord on the next quantity of time; and finally must be re-

B b

solved

solved on the third quantity of time, by descending to the next degree, either a tone or a semitone.

These operations depend on certain distinctions, to be observed in the divisions of time.

There are various species of musical time, but the divisions of each are comprehended in two or three equal quantities in a bar, or their multiples.

These bars are distinguished by a division of their equal parts into accented and unaccented. If a bar is composed of four equal quantities, the first and third are accented; the second and fourth are unaccented. If there are three quantities in a bar, the first only is accented, and the other two are unaccented. The preparing of a discord therefore must be on the unaccented part.

The same sound continues, or is a ligature, and forms the discord on the following accented part, after which it must descend to be resolved on the succeeding unaccented part, either a tone or a semitone.

Hence it appears, that all discords, prepared and resolved, require three quantities of time; the first quantity prepares the ear for its existence on the second, and the third satisfies the ear by the sweetness of its resolution.

This discord, like the minor seventh, has its origin in the *union* of the first and third terms; as may be seen at Pl. xxviii, in

in the fifth column, where D is its fundamental bass, with its perfect fifth and minor third; and again, at Pl. xxix, in the fifth column, where F is the fundamental bass, with its perfect fifth and major third.

As this discord now stands, we may observe that the three terms are employed in its formation, and that the second term is placed a fifth from each of the extremes, to which a major or minor third to the fundamental bass is added, which completes its harmonics.

The construction of these two ninths is in all respects the same, except the minor third to the first, and the major to the second; which not only makes this discord proper for the two scales, but also for the different parts of the scales.

By this we discover, that though its original is on a primary fundamental bass, yet it may exist on a secondary fundamental bass, and may therefore be applied not only to either of the three terms, but also to a fourth term, if properly prepared and resolved.

But as the character and genius of every discord can only be discovered by its motion and relation to the second term, it will be proper to compare this ninth with the minor seventh, to enable us to judge of its being a new species of discord, from its different preparations and resolutions, as well as from its motion and relation to the second term.

The ninth may be prepared by being a fifth, but not by being a third, to the fundamental bass.

The seventh is prepared by being a fifth or third to the fundamental bass.

The fundamental bass continues two quantities of time to receive the resolution of the ninth into the eighth.

The fundamental bass falls a fifth, or a third, to receive the resolution of the seventh into the third, or eighth.

The ninth may be applied to the first term, and after its resolution the second term may succeed.

The seventh cannot exist on a first term, without changing its character to a fourth term, and being obliged to move to other fourth terms, before it can arrive at the second term.

The ninth may be applied to the second term without changing its character, or the scale.

The seventh cannot be applied to a second term without changing both.

The ninth must always be prepared, unless united with the seventh, as a double compounded harmonic.

The seventh is frequently used in descending systems without being prepared.

These differences between the two discords must be owned to be very great, more especially as the motion of the fundamental bass, its character, and relation to the second term, are so particularly determined.

This is the great object to which all harmonics should be directed; and unless some distinction marks the phrase, the sentiment, and the distance of the fundamental bass, which leads

us to the principal generator, or second term, our pleasure must abate in proportion to the difficulties we have to encounter.

As all inversions teach us to diversify an original harmony, without deviating from the natural genius and character of the fundamental bass, I would recommend a constant attention to the fundamental bass, its motion and harmonics, particularly those which precede any species of discord: the same attention must be paid to those which succeed, and are the resolution of the discords; for although these two species of discord, the seventh and the ninth, should each be prepared by a fundamental bass descending a fifth, yet, by an attention to the phrase, to the sounds that move, and those which remain as ligatures, the species of discord will be easily determined. For if the third to the fundamental, which prepares the discord, is the ligature, we are sure the seventh is the discord; but if the fifth of that fundamental bass should be the ligature, the second species of discord, or ninth, must undoubtedly be applied.

Hence the discord is very early known by its preparation: as to its resolution, which is of such consequence to the phrase and the succeeding harmonics, we can be under no difficulty, if we remember that the natural motion of all original discords is to *descend* a tone or semitone, and observe the situation of the ninth, which by being placed one degree above the eighth of the fundamental bass, must, by a kind of necessity, move to it; but the contrary of this is true with respect to the seventh, which is placed a note under the octave of the fundamental bass, to which it cannot ascend, but must wait for another fundamental bass to be resolved into a third.

I must

I must also observe, that although the ninth is sometimes found on a secondary fundamental bass, yet it has a more natural connection with the three primary, and for that reason has a near relation to the second term. But it is not so with this seventh, which, as it constitutes the principal character of the fourth term, can never be otherwise applied, nor have any immediate relation to the second term.

There is one observation more I must make with respect to the ninth, which is, that there is no instance of its being applied, where we may not with propriety apply the simple harmonics, if it were not to vary the sentiment, by employing a double quantity of time. This enables us to suspend the ninth, by which means the succeeding sound arrives with a sweeter and more graceful effect.

I hope therefore it will not be thought improper to distinguish this discord as the *second species*, and to call it the suspension of the ninth. From the many distinctions made by comparing it with the seventh, if properly attended to, it must be impossible to mistake it, either with respect to its character, or the similitude of the figuring, as applied to the relative basses; for its preparation and resolution on fundamental basses are so peculiar to itself, as to prevent any possibility of its being taken for any other discord.

SECTION IX.

THE THIRD SPECIES OF DISCORD.

The Diatessaron, or perfect fourth, intended now for the *third* species of discord, has before been demonstrated as a perfect concord in the ratio of 3, 4.

How this duplicity of character can belong to the same interval is the subject of our present enquiry.

It must not be forgot, that the products of the first, second, and third terms, fixed and determined the place of every sound, which formed the scale, as well as all possible combination of consonant sounds; and that neither of these three terms alone could extend its powers to the formation of discord.

Their *Union* therefore in part, or in the whole, became absolutely necessary for the attainment of this great end; and at the same time demonstrates the perfection of a theory, which obliges us to draw every thing from its *original principles*.

This has in part been already explained in the two first species of discords, where the first term was added as a compound harmonic to the third term, and formed the minor seventh, or *first* species of discord; and by applying the third term, as harmonic to the first, the suspension of the ninth, or *second* species of discord was formed.

From

From the same original principles this *third* species of discord has its existence ; for by the union of the first and third terms we have the suspension of the fourth ; see Pl. XXVIII and XXIX ; which, by being united with the fifth to the fundamental bass, and subject to the laws before prescribed to the suspension of the ninth, must therefore be prepared and resolved.

The formation of this discord depends on the motion of the fundamental bass, with its simple harmonics rising a fifth on an accented part of the bar. This motion not only prepares the discord, but determines its combination to be an eighth, fifth, and fourth.

The fourth is prepared by the eighth of the preceding fundamental bass, and is substituted as a discord in the present combination, in lieu of the third, on an accented part of a bar, for one quantity of time. This suspension of the fourth must descend to the next degree, on the next unaccented part of the bar, to restore the third to its natural place, and complete the simple harmonics,

Thus a suspension appears to be only a power (subject to certain condition) of preventing, or keeping back, the motion of one sound for a time ; at the expiration of which it is put into its natural state, and then the harmonics are perfected.

This power, as applied to harmonics, contributes greatly to vary the sentiment, by employing three quantities of time
with

with the suspension, when two only would have been sufficient without it.

This consideration alone makes discords necessary, not only to distinguish the motion of the fundamental bass, but also to mark the time, and enforce the expression of the harmonics,

This suspension, or third species of discord, is formed in each diagram; (see Pl. xxviii and xxix, column 4.) and may therefore be resolved into a minor or major third, by descending a tone, or a semitone.

These two different resolutions will enable us to apply this discord to a fourth term, as well as to a third and second; but, from the nature of the scale, it can never be applied to a first term; for as that is the only fundamental bass below the second term, and has no perfect fifth below, it cannot, consistently with the scale and its character of a *first* term, admit any other discord but the second species, or ninth.

The second term is also limited to the second and third species of discords only; viz. the ninth and fourth. But the third and fourth terms may receive either of the three species of discords; and the first species may be either singly or doubly compounded, agreeably to the distinct character of the third and fourth terms before described.

As these limitations of the discords distinguish the different terms to which they are applied; so, by a comparison of their

preparations and resolutions, their several characters will be sufficiently known to determine the relation of any fundamental bass to the second term, or principal sound.

The 7th }
 The 9th } is prepared by a { 3^d, and the fundamental bass descends 5.
 The 4th } { 5th, and the fundamental bass descends 5.
 { 8th, and the fundamental bass ascends 5.

The 7th }
 The 9th } is resolved into a { 3^d, and the fundamental bass descends 5.
 The 4th } { 8th, and the fundamental bass continues.
 { 5^d, and the fundamental bass continues.

It has already been found, p. 178, that a fundamental bass preceding the third term a fifth higher, and descending a fifth to it, must be a secondary fundamental bass, or fourth term; so, if several fundamental basses precede the third term in this order of fifths, they must all be deemed so many fourth terms; for as they have nothing in their composition the least conclusive, either in relation to the phrase, or principal generator, till the third term appears, and discovers their remote relation to the second term, it follows, that the phrase must necessarily be lengthened, according to the number of fourth terms preceding the third and second term, which determines the phrase and the scale.

This third species of discord having been explained, together with its preparation by an eighth, and the motion of the fundamental bass ascending a fifth, I shall next describe it when prepared by a seventh. This preparation may be made by the seventh of a third or fourth term, by the fundamental
 bass

bass descending a fifth, and may be resolved into a third, either major or minor, the bass continuing on ; or may be resolved into another seventh if the root descends a fifth, and will have the best effect if on a third term, in order to proceed to the second term, on which it may exist as a fourth ; or it may be united with the whole harmonics of the third term, which will make a ninth a major seventh and fourth, and be resolved into the simple harmony of the second term on the next quantity of time.

The different positions of the harmonics of this third species of discord are many, but with respect to the figuring there can be only three ; viz. $\frac{5}{4}$, $\frac{5}{2}$, $\frac{7}{4}$; and, as was before observed, by a proper attention to the motion of the fundamental bass that precedes this discord, and discovers its preparation by the fundamental bass rising five, there will be found so great a difference from a discord prepared by one falling five, as must remove every appearance of difficulty.

From what has been above explained concerning the three species of discord, we find that they have received their original existence from the *union* of the *first* and *third* terms ; that by the motion of the fundamental bass, which determines their preparations and resolutions, we discover their true character, in what particulars they differ from each other, and their relation to the second term ; and that they are original, because they comprehend all other dissonances whatever by their inversions, or the different positions of the harmonic basses, without altering the rules before prescribed for the motion of the fundamental bass.

Before I draw to a conclusion, I would take notice of one very great mistake made by modern theorists, which, if they had determined the fact by the ratio, could not have escaped them. We meet with it in Kircher's third book, page 145, and in Zarlino, page 363, in the third book of his *Institutioni*; the same has been adopted by all succeeding writers. What I mean is their description of the enharmonic gender.

They have made the first interval in the tetrachord of the enharmonic gender a semitone minor, as B, B \times , C, and E; instead of the enharmonic diesis, which is B, C \flat , C \sharp , E. The deception of marking B with a single cross, instead of a double one, is equally exceptionable. By this mistake they have placed the enharmonic diesis the second interval in the tetrachord, when it ought to have been the first. But to prove this beyond a doubt, the tetrachord in question is in the first diagram, in which there is no B \sharp . See the harmonic tables, Pl. xiv, where B 128 in the triple, when compared with C \flat in the quintuple progression, root 25, is the enharmonic diesis in its true ratio $\frac{128}{125}$; to which if a semitone minor is added, $\frac{25}{24}$, they will make a semitone major, and the residue of the tetrachord will be a ditone, or third major, $\frac{5}{4}$, an imperfect concord, not peculiar to the enharmonic, as the same interval is found in the diatonic gender, in which the third minor is the incomposite interval, or residue of the chromatic gender.

The

The CONCLUSION.

HAVING considered at large in the *First* Part of this Work the Principles of the Grecian Doctrine of the Harmonica, as maintained by the *Aristoxenians*, and in this *Second* Part as supported by the *Pythagoreans*, in the Doctrine of the Ratio, it may be useful to make a few summary reflections on these two very extraordinary theories. — The former, maintained by Aristoxenus, Euclid, and Bacchius Senior, cannot be examined with too great attention, as it abounds with whatever is curious and interesting to the subject. The order and disposition of the several systems, particularly the tetrachord, from which the scales are formed, together with the number of flat and sharp dieses, by which the minor and major scales are completed; above all the seven species of the diapason, in which the tetrachords are comprehended, and regulated by the conjunction and disjunction; are among the many instances of the perfection of this theory.

The mese, or tone of disjunction, is of the greatest consequence; for it not only prevents two eights, two fifths, or two major thirds in succession, but stops us from moving into another scale, unless by the intervention of an additional base as a temperament, which marks and discovers the alteration necessary for a new scale. — The genders supply the imperfect consonances; viz. the major third in the enharmonic, and the minor in the chromatic gender; each of which being taken from the diapason, there remain the two species of sixths; in the same manner as the diapente, added to the diatessaron, completes the magnitude of diapason. By the same method the diatonic moll forms the interval of a superfluous second, which,
together

together with the extreme flat seventh, completes the diapason.

The great advantage of discovering these intervals contributes to the perfecting of this theory; for notwithstanding the seven species of diapason contained them all, yet their formation would have been attended with great difficulty, and their magnitude must sometimes have been exceptionable; as from B to D in the first diagram, and from F to D in the second, are each a minor third deficient by a comma: yet it must be observed, that although the Aristoxenians did not admit of the ratio in their theory, their division of intervals was strictly conformable to it. Thus the hemitone minor and major, which form a part of the chromatic gender, when added together are the sum of the tone minor, and not the tone major. As to the *ipsis*, or smaller intervals, they are so necessary to the mutations, that nothing could be perfect without them; and though they may be taken out of the different species of diapason, yet the genders regulate and determine the magnitude of all the less intervals, with much greater ease and certainty.

If again we examine the rules laid down for the mutations, they are simple and easy to be understood; for they are comprehended in the perfect consonant mutation, to ascend or descend a fifth; the imperfect mutation, to ascend or descend a third; and the dissonant mutation, to ascend or descend a tone. These mutations are regulated by adding one *diefis*, or by taking one away from the first proposed number; by which means the mode is formed and supplied with all its varieties.

In

In the First part of this Work I did not treat of the Discords ; not that I was without materials ; but as a Second Part, supported by the ratio, has succeeded to the former, in that I have explained the three species of original discords, agreeably to the numbers applied by Pythagoras to discover the true ratio of concord and discord ; viz. 6, 8, 9, 12. I was desirous of doing honour to his memory, as the first we know of, who discovered the magnitude of musical intervals by the ratio ; which very properly succeeds the two diagrams of Gaudentius, having the same principles applied to each.

But here I must beg leave to pause one moment, to pay a tribute of gratitude most justly due to the merit of that great and excellent theorist and professor of Music, the learned Dr. *Pepusch* ; from whom, in an early part of life, I received the first principles of the application of numbers to musical intervals. As I loved and honoured him while living, so I greatly lamented his death, which happened at a time when I stood most in need of his assistance. His church music, preserved in the Ancient Academy, is his best eulogy as a Composer.

While I pay this tribute due to the memory of a departed friend, I am happy to embrace the opportunity of acknowledging the very friendly assistance I have received in this work from the *Rev. J. Trebeck*, and the *Rev. G. S. Townley*.

I shall

I shall now make some observations on the *Second* Part, or the doctrine of the Ratio, maintained by Gaudentius, and by Pythagoras.

The great advantage of discovering the magnitude of all musical intervals by the Ratio cannot but be agreeable to men of science and lovers of truth. The Ratio has brought out of obscurity the two diagrams of Gaudentius and removed several objections made by writers of the first reputation.

These diagrams consisting of tones major and limmas occasioned the first and principal objection to the Grecian harmonica, as seeming destitute of the imperfect concords, without which, it is impossible to have harmony, or even melody ; which last the Grecians are acknowledged to have had in the greatest perfection. With respect to the imperfect concords, it is very extraordinary that no writer, whom I have seen, ever attempted to discover them in the genders : for as to the seven species of the diapason, from whence the genders are formed, as well as all other musical intervals, it was not possible to extract them from thence, unless the position and harmonious constitution of those diapasons, together with their *Inversion*, had been first discovered, and perfectly understood by the moderns.

With respect to the objection above-mentioned to the diagrams of Gaudentius, it was so true, that nothing could remove it but the application of numbers, or the harmonic principles : these were no sooner applied, than the clouds of darkness disappeared, and the *great*, the *perfect* and *immutable* systems of the diagrams, instead of being (as was imagined) a scale of
sounds

sounds intended for the voice, are found to be roots, principal tones, or keys of several scales, formed into systems, in the ratio of tones major and limmas. However extraordinary these diagrams may appear, yet (as before observed) the same exist in our own theory; for if the principal tones of different scales be placed in the same order of degrees, they will be each to the next in the ratio of tone major or limma. Hence all the objections made to the Grecian harmonica on account of these Diagrams, must vanish, or must in an equal degree affect our own: which would be a task too difficult for any one to prove.

It is very extraordinary that authors of the first reputation should condemn the Grecians for having no imperfect concords, when they are so easily discovered in the three genders, even according to their own definitions. But authors are sometimes tempted to take upon trust, whatever they meet with in the works of men of character, rather than give themselves the trouble to discover the truth.

Five has been rejected as one of the primes, though the enharmonic diesis (which they all agree is a part of that gender) cannot be formed without it. For as 125 is a multiple of 5, in the quintuple proportion, it must therefore be one of the primes. The quintuple proportion is sometimes useful in the mutations; but as it is incapable of a progression, it must always be succeeded by the triple proportion. Though in neither of these proportions can the mutations of the enharmonic diesis, or of the limma, be found: the former is not applied in combination, nor mentioned by Euclid, when he treats of the mutations; which is a confirmation that it was only necessary to determine the exact

D d

magnitude

magnitude of the ditone, or major third, in the enharmonic gender; as the chromatic was with respect to the minor third in the chromatic gender. The limma cannot be applied in combination, nor in the mutations, and therefore can be found only in the diagrams, being absolutely necessary for their formation, as systems of principal sounds.

I would here observe, that in the course of this work it has been shewn, that the fundamental bass cannot without a temperament ascend a tone or semitone, nor descend a tone or semitone; yet exceptions to these general laws are found in modern compositions in distinct phrases, in the Cadenza fugita, and in extra-harmonic sounds, which balk the cadence, and interrupt the regular progression of the fundamental bass.

These, and many other remarks which have been made in the course of this work, seemed to me necessary to remove every objection that could materially affect any essential part of the *Grecian Harmonica*: in which we discover the laws of science so intimately united and blended with those of nature, that the least deviation from the latter must in an equal degree be an injury to the simplicity of the former. That simplicity excludes the necessity of a profound knowledge in Geometry, in order to discover the original of the minor and major scales, of concords, discords, genders, modes, and other parts of the ancient Harmonica, which have long been perplexed and obscured by spheres, squares, cylinders, cones, and other geometrical figures, that have contributed little or nothing to the improvement of the practical Musician.

The

The *Aristoxenians* discovered the Agreement of concord and discord, and determined the magnitude of musical intervals by the laws of their own natural Feelings.

The *Pythagoreans* applied themselves to discover the *Ground* of those laws; that by some leading and universal principle they might preserve the voice of nature in its most pure and original state. Hence they found that all melodies originated in, and were regulated by, the fundamental bass. Hence their diagrams were confined to a geometric proportion in the ratio of 3 : and the harmonious construction of the tetrachord, the conjunction and disjunction of tetrachords, the minor and major scales, the flat and sharp dièses, were all regulated and determined by this first unerring principle, a progression in the ratio of 3.

Here I cannot but take notice how generally this universal principle has regulated all musical compositions, even to this day; and in many instances, where we cannot suppose it to be understood. Guido has placed his three hexachords (as was before observed) in the same ratio: and if we examine the three cliffs of the moderns F, C, and G, and the tuning of many stringed instruments, as in particular the Violin, Tenor, and Violoncello, their relation to each other is the same. Moreover the several species of discord (formed out of three terms by the union of the first and third terms in *combination*, whose motion could not be permitted in *succession*) their preparations and resolutions, the mutations with the same species of thirds, are all governed by the same ratio. Could all this be by Chance? rather, is there not an absolute necessity, that All shall comply

D d 2

with

with principles founded in Nature? to whose laws we must be obedient, whether they are thoroughly understood or not.

Thus this wonderful theory of the Grecians, established on the most solid foundation, proceeds from a given sound, or principal generator, and advances with its creative powers, to generate its two products, the first and third terms, from whence arise all consonance and dissonance, scales and systems, major and minor, an encreasing series of flats and sharps, the tone major and minor, the different species of semitones, and all other intervals adapted to the mutations; the three genders, diatonic, chromatic, and enharmonic, the triple and quintuple progression of the fundamental bass, and whatever else is necessary to adorn, enrich and perfect the Science of Harmony.

It is almost impossible to enumerate all the advantages with respect to musical information and a profound knowledge of harmony, contained in the *Grecian Harmonica*. The simplicity and agreement of its several parts, established on laws that coincide with natural feelings and remain unalterable, are considerations sufficient to recommend it to all those who either wish to excel as Composers, or are desirous of attaining a *true knowledge* of the
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TO THE
THEORY
OF
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N.B. By a mistake in *numbering* the Plates, No. XI. was omitted; in consequence of which, Plate X. is immediately followed by Plate XII.

Page.	Line.		
24,	1,	for <i>of Harmony,</i>	read <i>of the Harmonica.</i>
27,	3,	for <i>diefes,</i>	read <i>diefis.</i>
31,	22,	for <i>diefes,</i>	read <i>diefis.</i>
50,	20,	for <i>Pate,</i>	read <i>Plate.</i>
72,	6,	for <i>different,</i>	read <i>sufficient.</i>
100,	14,	for <i>next,</i>	read <i>less.</i>
117,	6,	for <i>prog sion,</i>	read <i>progression.</i>
131,	3,	for <i>Guadentius,</i>	read <i>Gaudentius.</i>
167,	10,	for <i>them otion,</i>	read <i>the motion.</i>
169,	7,	for <i>Pl. vii.</i>	read <i>Pl. xxii.</i>
173,	1,	for <i>the seventh,</i>	read <i>this seventh.</i>

MAJOR SYSTEM.

MINOR SYSTEM.

The diagram illustrates the Grecian Scale, divided into Major and Minor systems. The Major system is shown at the top, and the Minor system is shown at the bottom. The scale is represented by two staves, each with a treble clef. The notes are labeled with Greek letters: Α (Alpha), Β (Beta), Γ (Gamma), Δ (Delta), Ε (Epsilon), Ζ (Zeta), Η (Eta), Θ (Theta), Ι (Iota), Κ (Kappa), Λ (Lambda), Μ (Mu), Ν (Nu), Ξ (Xi), Ο (Omicron), Π (Pi), Ρ (Rho), Σ (Sigma), Τ (Tau), Υ (Upsilon), Φ (Phi), Χ (Chi), Ψ (Psi), Ω (Omega). The intervals between the notes are labeled as follows:

- Major System Intervals:**
 - Α to Β: *Proslambanomenos*
 - Β to Γ: *Hypate Hypaton*
 - Γ to Δ: *Parhypate Hypaton*
 - Δ to Ε: *Lichanos Hypaton Diatonos*
 - Ε to Ζ: *Hypate Meson*
 - Ζ to Η: *Parhypate Meson*
 - Η to Θ: *Lichanos Meson Diatonos*
 - Θ to Ι: *Mese*
 - Ι to Κ: *Trite Synemmenon*
 - Κ to Λ: *Paranete Synemmenon Diatonos*
 - Λ to Μ: *Nete Synemmenon*
 - Μ to Ν: *Paramese*
 - Ν to Ξ: *Trite Diezeugmenon*
 - Ξ to Ο: *Paranete Diezeugmenon Diatonos*
 - Ο to Π: *Nete Diezeugmenon*
 - Π to Ρ: *Trite Hyperbolaeon*
 - Ρ to Σ: *Paranete Hyperbolaeon Diatonos*
 - Σ to Τ: *Nete Hyperbolaeon*
- Minor System Intervals:**
 - Α to Β: *Proslambanomenos*
 - Β to Γ: *Hypate Hypaton*
 - Γ to Δ: *Parhypate Hypaton*
 - Δ to Ε: *Lichanos Hypaton Diatonos*
 - Ε to Ζ: *Hypate Meson*
 - Ζ to Η: *Parhypate Meson*
 - Η to Θ: *Lichanos Meson Diatonos*
 - Θ to Ι: *Mese*
 - Ι to Κ: *Trite Synemmenon*
 - Κ to Λ: *Paranete Synemmenon Diatonos*
 - Λ to Μ: *Nete Synemmenon*
 - Μ to Ν: *Paramese*
 - Ν to Ξ: *Trite Diezeugmenon*
 - Ξ to Ο: *Paranete Diezeugmenon Diatonos*
 - Ο to Π: *Nete Diezeugmenon*
 - Π to Ρ: *Trite Hyperbolaeon*
 - Ρ to Σ: *Paranete Hyperbolaeon Diatonos*
 - Σ to Τ: *Nete Hyperbolaeon*

The diagram also includes labels for the tetrachords:

- Tetrachord Hypaton:** Α, Β, Γ, Δ
- Tetra. Meson:** Ε, Ζ, Η, Θ
- Tetra. Synemmenon:** Ι, Κ, Λ, Μ
- Tetra. Diezeugmenon:** Ν, Ξ, Ο, Π
- Tetra. Hyperbolaeon:** Ρ, Σ, Τ, Υ

Plate II. The seven Species of Diapafon.

First Diagram.

Fig. I.

1 Mixolydian.

2 Lydian.

3 Phrygian.

4 Dorian.

5 Hypolydian.

6 Hypophrygian.

7 Hypodorian.

Fig. II.

1 H.

2 N.

3 L.

4 J.

5 P.

6 M.

7 K. +

EUCLID'S DIVISIONS of the TETRACHORD.

Plate IV.

FIRST DIAGRAM.

DIAPASON.

TETRACHORD.

K. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$

L. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$

N. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **ENHARMONIC.**

P. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **SESQUIALTER CHROMATIC.**

R. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **TONLÆUM CHROMATIC.**

S. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **DIATONIC MOLL.**

T. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **DIATONIC SYNTONE.**

W. $\text{C} \text{ D} \text{ E} \text{ F} \text{ G} \text{ A} \text{ B} \text{ C}$ = 30 **CHROMATIC MOLL.**

X.

enharmonic diesis	semitone minor	semitone major	7 th major
tone	7 th minor	superfluous 2	deficient 7 th
deficient 3 ^d	superfluous 6 th	minor 3 rd	major 6 th
major 3 ^d	minor 6 th	deficient 4 th	superfluous 5 th
perfect 4 th	perfect 5 th	tritone or greater 4 th	semidiapente.

EUCLID'S DIVISIONS of the TETRACHORD.

SECOND DIAGRAM.

Plate V.

DIAPASON.

TETRACHORD.
TETRACHORD. +

= 30 ENHARMONIC.

= 30 SESQUIALTER CHROMATIC.

= 30 TONIAEUM CHROMATIC.

= 30 DIATONIC MOLL.

= 30 DIATONIC SYNTONE.

= 30 CHROMATIC MOLL.

Z

enharmonic diesis. femitonem minor. femitonem major. 7th major.

tone. 7th minor. superfluous 2nd deficient 7th

deficient 3^d superfluous 6th minor 3^d major 6th

major 3^d minor 6th deficient 4th superfluous 5th

perfect 4th perfect 5th femidiapente. tritone or greater 4th

Plate VI. Inversion of the Seven Species of Diapason.
SECOND DIAGRAM.

Fig. 1 First Diagram

Fig. 2

Second Diagram

Fig. 3

Fig. 4

The Seven Species of Diapason.

Plate VII

T.

SECOND DIAGRAM. U.

U.

First system of musical notation, labeled I on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Second system of musical notation, labeled II on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Third system of musical notation, labeled III on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Fourth system of musical notation, labeled IV on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Fifth system of musical notation, labeled V on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Sixth system of musical notation, labeled VI on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Seventh system of musical notation, labeled VII on both staves. The upper staff (T.) contains a treble clef and a series of notes with a '+' sign above the fourth measure. The lower staff (U.) contains a bass clef and a series of notes with a '+' sign above the fourth measure. The notation includes various accidentals and a key signature of one flat.

Scale VIII. Scales with Major and Minor thirds extracted from the Harmonics of the Three Terms.

7 Sharps

6 Sharps V Species

5 Sharps II Species

4 Sharps VI Species

3 Sharps III Species

2 Sharps VII Species

1 Sharp IV Species

Natural First Species of Diapason descending and ascending.

1. 2. 3. 4. Term.

Scales with Major and Minor thirds extracted from the Harmonics of the Three Terms.

1 Flat IV Species 1st 2nd 3rd Term.

2 Flats VII Species

3 Flats III Species

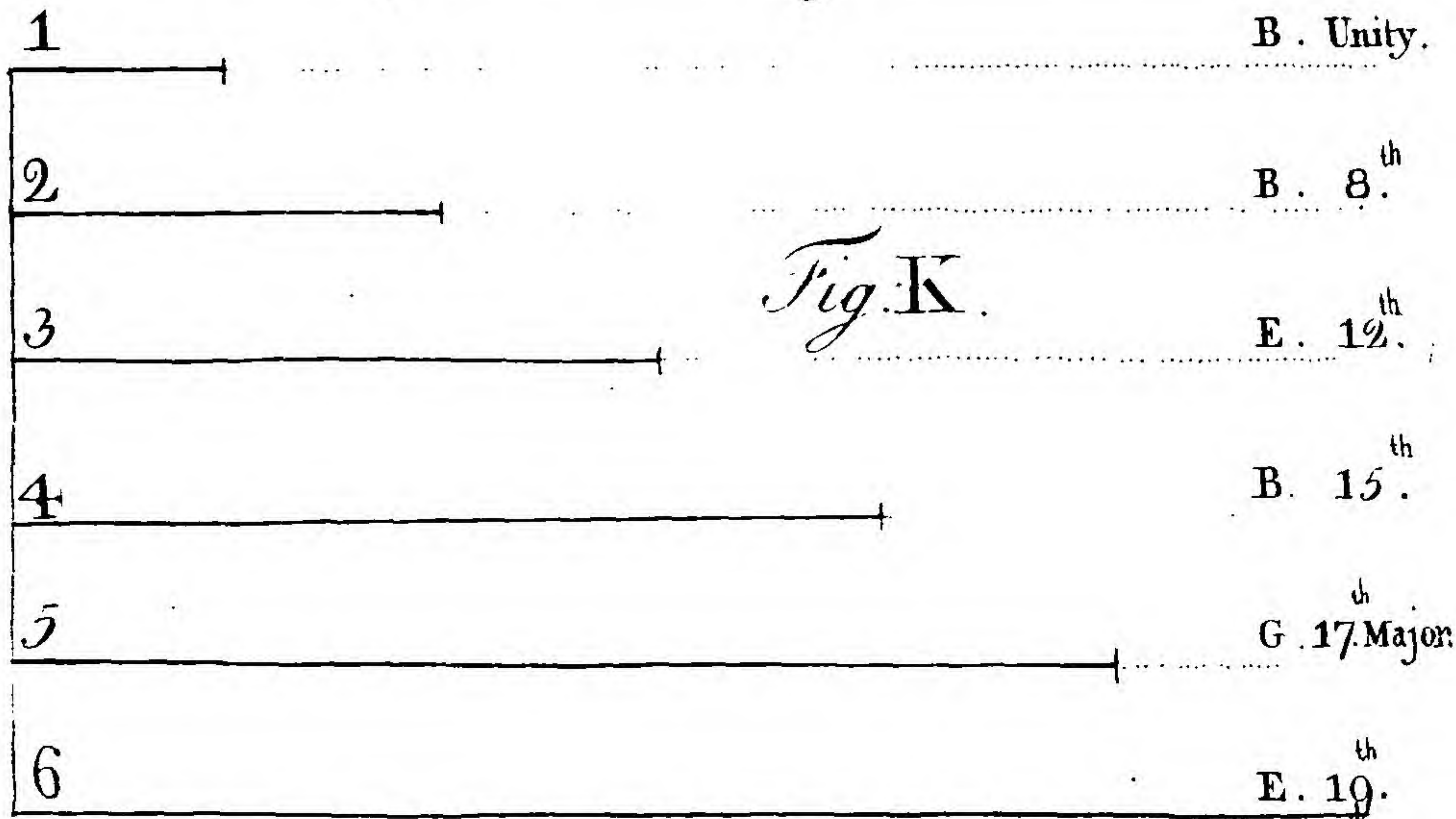
4 Flats VI Species

5 Flats II Species

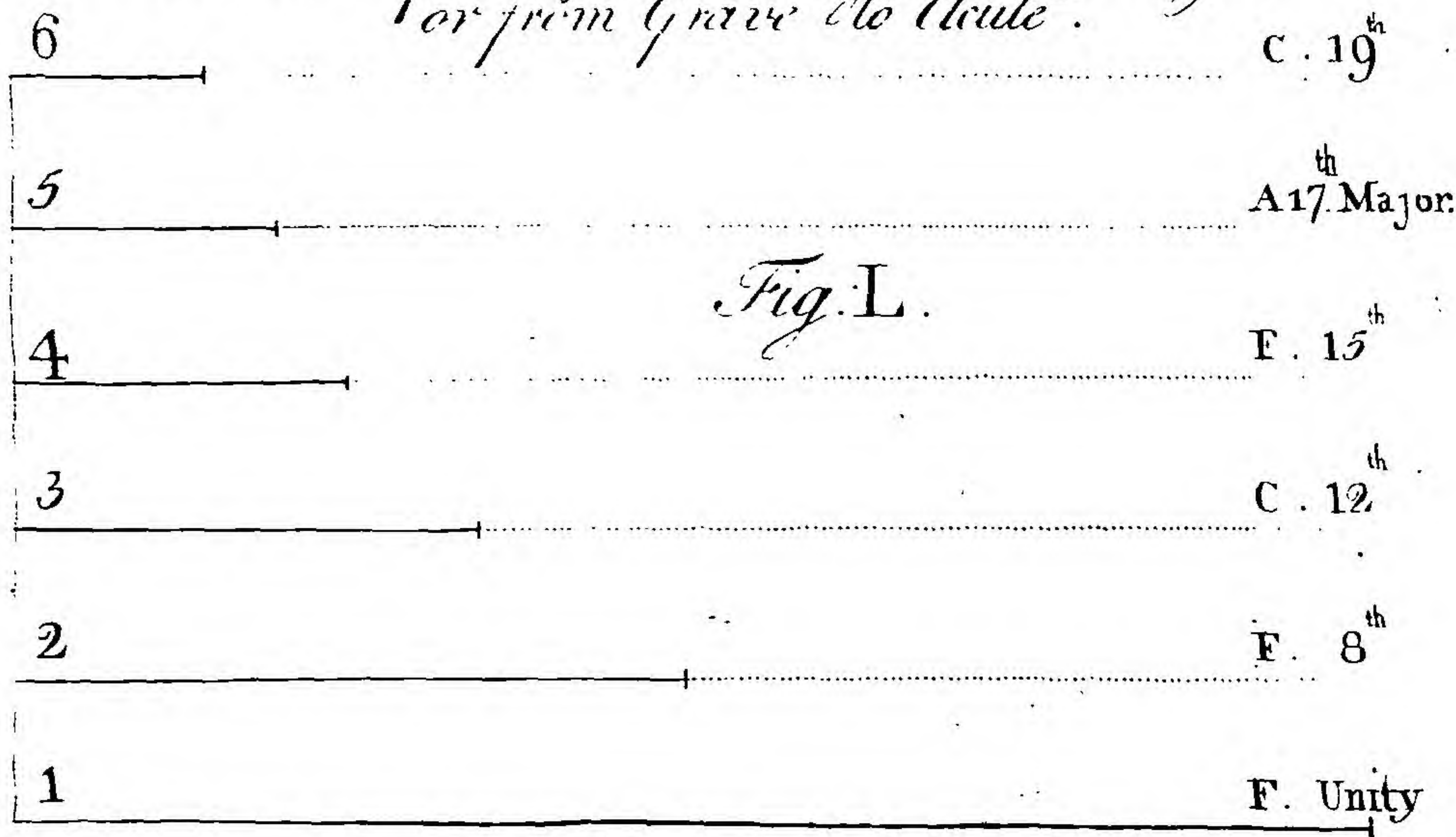
6 Flats V Species

7 Flats

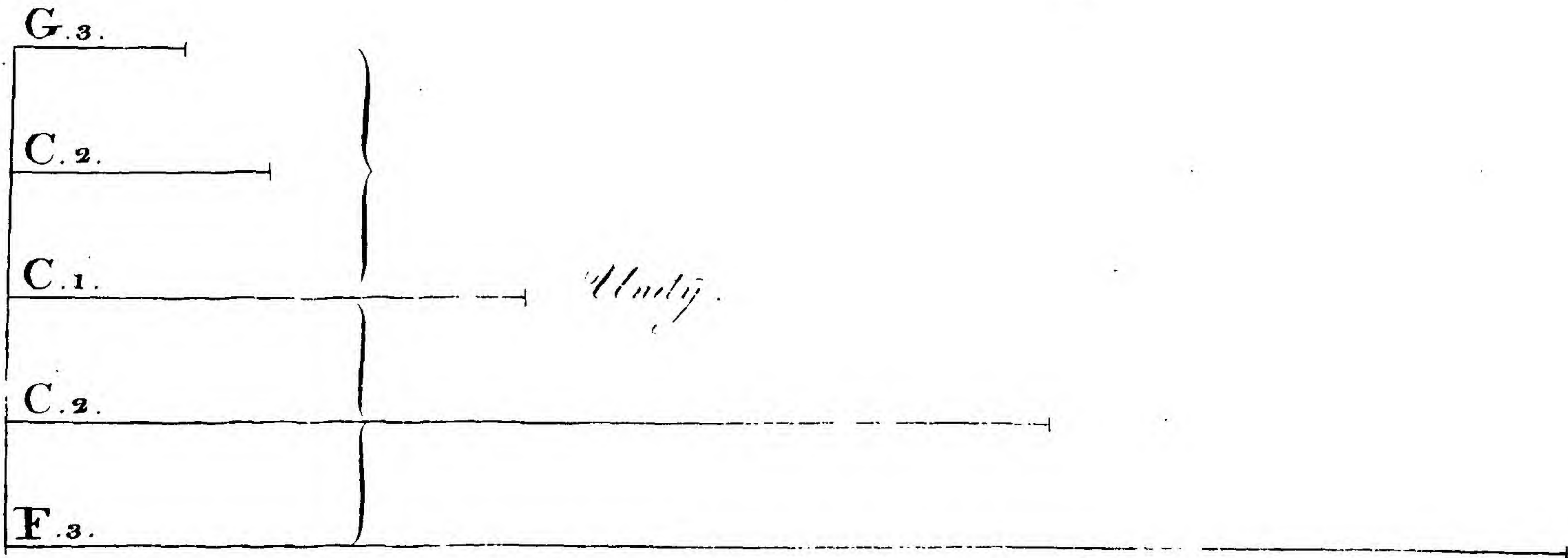
*Lengths of Strings descending from Unity,
or from Acute to Grave.*



*Vibrations of Strings ascending from Unity,
or from Grave to Acute.*



Vibrations from Unity.



Lengths of Strings from Unity.

HARMONIC TABLE.
SECOND DIAGRAM.

ROOT 27 th HARMONICS.	8	D	216	432	864	1728	3456	6912	13824	27648	55296	110592
	6	A	162	324	648	1296	2592	5184	10368	20736	41472	82944
	5	F#	135	270	540	1080	2160	4320	8640	17280	34560	69120
	4	D	108	216	432	864	1728	3456	6912	13824	27648	55296
	3	A	81	162	324	648	1296	2592	5184	10368	20736	41472
	2	D	54	108	216	432	864	1728	3456	6912	13824	27648
	1	D	27	54	108	216	432	864	1728	3456	6912	13824
ROOT 9 th HARMONICS.	8	G	72	144	288	576	1152	2304	4608	9216	18432	36864
	6	D	54	108	216	432	864	1728	3456	6912	13824	27648
	5	B	45	90	180	360	720	1440	2880	5760	11520	23040
	4	G	36	72	144	288	576	1152	2304	4608	9216	18432
	3	D	27	54	108	216	432	864	1728	3456	6912	13824
	2	G	18	36	72	144	288	576	1152	2304	4608	9216
	1	G	9	18	36	72	144	288	576	1152	2304	4608
ROOT 3 rd HARMONICS.	8	C	24	48	96	192	384	768	1536	3072	6144	12288
	6	G	18	36	72	144	288	576	1152	2304	4608	9216
	5	E	15	30	60	120	240	480	960	1920	3840	7680
	4	C	12	24	48	96	192	384	768	1536	3072	6144
	3	G	9	18	36	72	144	288	576	1152	2304	4608
	2	C	6	12	24	48	96	192	384	768	1536	3072
	1	C	3	6	12	24	48	96	192	384	768	1536
ROOT 1 st HARMONICS.	8	F	8	16	32	64	128	256	512	1024	2048	4096
	6	C	6	12	24	48	96	192	384	768	1536	3072
	5	A	5	10	20	40	80	160	320	640	1280	2560
	4	F	4	8	16	32	64	128	256	512	1024	2048
	3	C	3	6	12	24	48	96	192	384	768	1536
	2	F	2	4	8	16	32	64	128	256	512	1024
	1	F	1	2	4	8	16	32	64	128	256	512

continued

SECOND DIAGRAM. HARMONIC TABLE

continued

Root 2187 & ITS HARMONICS.	8	F#	17496	34992	69984	139968	279936	559872	1119744
	6	C#	13122	26244	52488	104976	209952	419904	839808
	5	A#	10935	21870	43740	87480	174960	349920	699840
	4	F#	8748	17496	34992	69984	139968	279936	559872
	3	C#	6561	13122	26244	52488	104976	209952	419904
	2	F#	4374	8748	17496	34992	69984	139968	279936
	1	F#	2187	4374	8748	17496	34992	69984	139968
Root 729 & ITS HARMONICS.	8	B	5832	11664	23328	46656	93312	186624	373248
	6	F#	4374	8748	17496	34992	69984	139968	279936
	5	D#	3645	7290	14580	29160	58320	116640	233280
	4	B	2916	5832	11664	23328	46656	93312	186624
	3	F#	2187	4374	8748	17496	34992	69984	139968
	2	B	1458	2916	5832	11664	23328	46656	93312
	1	B	729	1458	2916	5832	11664	23328	46656
Root 243 & ITS HARMONICS.	8	E	1944	3888	7776	15552	31104	62208	124416
	6	B	1458	2916	5832	11664	23328	46656	93312
	5	G#	1215	2430	4860	9720	19440	38880	77760
	4	E	972	1944	3888	7776	15552	31104	62208
	3	B	729	1458	2916	5832	11664	23328	46656
	2	E	486	972	1944	3888	7776	15552	31104
	1	E	243	486	972	1944	3888	7776	15552
Root 81 & ITS HARMONICS.	8	A	648	1296	2592	5184	10368	20736	41472
	6	E	486	972	1944	3888	7776	15552	31104
	5	C#	405	810	1620	3240	6480	12960	25920
	4	A	324	648	1296	2592	5184	10368	20736
	3	E	243	486	972	1944	3888	7776	15552
	2	A	162	324	648	1296	2592	5184	10368
	1	A	81	162	324	648	1296	2592	5184

continued

Second Diagram Harmonic Table.

continued.

Root 177147.	8	A ♯	1417176
	6	E ♯	1062882
	5	C ♯♯	885535
	4	A ♯	708588
	3	E ♯	531441
	2	A ♯	354294
	1	A ♯	177147
Root 59049.	8	D ♯	452392
	6	A ♯	354294
	5	F ♯♯	295245
	4	D ♯	236196
	3	A ♯	177147
	2	D ♯	118098
	1	D ♯	59049
Root 19683.	8	G ♯	157464
	6	D ♯	118098
	5	B ♯	98415
	4	G ♯	78732
	3	D ♯	59049
	2	G ♯	39366
	1	G ♯	19683
Root 6561.	8	C ♯	52488
	6	G ♯	39366
	5	E ♯	32805
	4	C ♯	26244
	3	G ♯	19683
	2	C ♯	13122
	1	C ♯	6561

HARMONIC TABLE.
FIRST DIAGRAM.

HARMONICS of Root 1.	1	B	1	2	4	8	16	32	64	128	256	512
	2	B	2	4	8	16	32	64	128	256	512	1024
	3	E	3	6	12	24	48	96	192	384	768	1536
	4	B	4	8	16	32	64	128	256	512	1024	2048
	5	G	5	10	20	40	80	160	320	640	1280	2560
	6	E	6	12	24	48	96	192	384	768	1536	3072
	8	B	8	16	32	64	128	256	512	1024	2048	4096
	1	E	3	6	12	24	48	96	192	384	768	1536
Root 3.	2	E	6	12	24	48	96	192	384	768	1536	3072
	3	A	9	18	36	72	144	288	576	1152	2304	4608
	4	E	12	24	48	96	192	384	768	1536	3072	6144
	5	C	15	30	60	120	240	480	960	1920	3840	7680
	6	A	18	36	72	144	288	576	1152	2304	4608	9216
	8	E	24	48	96	192	384	768	1536	3072	6144	12288
	1	A	9	18	36	72	144	288	576	1152	2304	4608
	2	A	18	36	72	144	288	576	1152	2304	4608	9216
Root 9.	3	D	27	54	108	216	432	864	1728	3456	6912	13824
	4	A	36	72	144	288	576	1152	2304	4608	9216	18432
	5	F	45	90	180	360	720	1440	2880	5760	11520	23040
	6	D	54	108	216	432	864	1728	3456	6912	13824	27648
	8	A	72	144	288	576	1152	2304	4608	9216	18432	36864
	1	D	27	54	108	216	432	864	1728	3456	6912	13824
	2	D	54	108	216	432	864	1728	3456	6912	13824	27648
	3	G	81	162	324	648	1296	2592	5184	10368	20736	41472
Root 27.	4	D	108	216	432	864	1728	3456	6912	13824	27648	55296
	5	Bb	135	270	540	1080	2160	4320	8640	17280	34560	69120
	6	G	162	324	648	1296	2592	5184	10368	20736	41472	82944
	8	D	216	432	864	1728	3456	6912	13824	27648	55296	110592

HARMONIC TABLE. FIRST DIAGRAM.

continued

HARMONICS of Root 81	1	G	81	162	324	648	1296	2592	5184
	2	G	162	324	648	1296	2592	5184	10368
	3	C	243	486	972	1944	3888	7776	15552
	4	G	324	648	1296	2592	5184	10368	20736
	5	E _b	405	810	1620	3240	6480	12960	25920
	6	C	486	972	1944	3888	7776	15552	31104
	8	G	648	1296	2592	5184	10368	20736	41472
Root 243.	1	C	243	486	972	1944	3888	7776	15552
	2	C	486	972	1944	3888	7776	15552	31104
	3	F	729	1458	2916	5832	11664	23328	46656
	4	C	972	1944	3888	7776	15552	31104	62208
	5	A _b	1215	2430	4860	9720	19440	38880	77760
	6	F	1458	2916	5832	11664	23328	46656	93312
	8	C	1944	3888	7776	15552	31104	62208	124416
Root 729.	1	F	729	1458	2916	5832	11664	23328	46656
	2	F	1458	2916	5832	11664	23328	46656	93312
	3	B _b	2187	4374	8748	17496	34992	69984	139968
	4	F	2916	5832	11664	23328	46656	93312	186624
	5	D _b	3645	7290	14580	29160	58320	116640	233280
	6	B _b	4374	8748	17496	34992	69984	139968	279936
	8	F	5832	11664	23328	46656	93312	186624	373248
Root 2187.	1	B _b	2187	4374	8748	17496	34992	69984	139968
	2	B _b	4374	8748	17496	34992	69984	139968	279936
	3	E _b	6561	13122	26244	52488	104976	209952	419904
	4	B _b	8748	17496	34992	69984	139968	279936	559872
	5	G _b	10935	21870	43740	87480	174960	349920	699860
	6	E _b	13122	26244	52488	104976	209952	419904	839808
	8	B _b	17496	34992	69984	139968	279936	559872	1119744

continued

First Diagram. Harmonic Table continued

HARMONICS OF ROOT 6561	1	E♭	6561
	2	E♭	13122
	3	A♭	19683
	4	E♭	26244
	5	C♭	32805
	6	A♭	39366
	8	E♭	52488
ROOT 19683	1	A♭	19683
	2	A♭	39366
	3	D♭	59049
	4	A♭	78732
	5	F♭	78415
	6	D♭	115098
	8	A♭	157464
ROOT 59049	1	D♭	59049
	2	D♭	115098
	3	G♭	177147
	4	D♭	236196
	5	B♭♭	295245
	6	G♭	354294
	8	D♭	452392
ROOT 177147	1	G♭	177147
	2	G♭	354294
	3	C♭	531441
	4	G♭	708588
	5	E♭♭	885535
	6	C♭	1062882
	8	G♭	1417176

FIRST DIAGRAM.
QUINTUPLE.

Plate XV.

Root 5.	4	G	20	40	80	160	320	640	1280	2560
		E \sharp	24	48	96	192	384	768	1536	3072
	5	E \flat	25	50	100	200	400	800	1600	3200
	6	C	30	60	120	240	480	960	1920	3840
Root 15.	4	C	60	120	240	480	960	1920	3840	7680
		A \sharp	72	144	288	576	1152	2304	4608	9216
	5	A \flat	75	150	300	600	1200	2400	4800	9600
	6	F	90	180	360	720	1440	2880	5760	11520
Root 45.	4	F	180	360	720	1440	2880	5760	11520	23040
		D \sharp	216	432	864	1728	3456	6912	13824	27648
	5	D \flat	225	450	900	1800	3600	7200	14400	28800
	6	B \flat	270	540	1080	2160	4320	8640	17280	34560
Root 135.	4	B \flat	540	1080	2160	4320	8640	17280	34560	69120
		G \sharp	648	1296	2592	5184	10368	20736	41472	82944
	5	G \flat	675	1350	2700	5400	10800	21600	43200	86400
	6	E \flat	810	1620	3240	6480	12960	25920	51840	103680
Root 405.	4	E \flat	1620	3240	6480	12960	25920	51840	103680	207360
		C \sharp	1944	3888	7776	15552	31104	62208	124416	248832
	5	C \flat	2025	4050	8100	16200	32400	64800	129600	259200
	6	A \flat	2430	4860	9720	19440	38880	77760	155520	311040
Root 1215.	4	A \flat	4860	9720	19440	38880	77760	155520	311040	622080
		F \sharp	5832	11664	23328	46656	93312	186624	373248	746496
	5	F \flat	6075	12150	24300	48600	97200	194400	388800	777600
	6	D \flat	14610	29220	58440	116880	233760	467520	935040	1870080

FIRST DIAGRAM. QUINTUPLE.

Root 25.	4	E♭	100	200	400	800	1600	3200	6400
		C♯	120	240	480	960	1920	3840	7680
	5	C♭	125	250	500	1000	2000	4000	8000
	6	A♭	150	300	600	1200	2400	4800	9600
Root 75.	4	A♭	300	600	1200	2400	4800	9600	19200
		F♯	360	720	1440	2880	5760	11520	23040
	5	F♭	375	750	1500	3000	6000	12000	24000
	6	D♭	450	900	1800	3600	7200	14400	28800
Root 225.	4	D♭	900	1800	3600	7200	14400	28800	57600
		B♭	1080	2160	4320	8640	17280	34560	69120
	5	B♭♭	1125	2250	4500	9000	18000	36000	72000
	6	G♭	1350	2700	5400	10800	21600	43200	86400
Root 675.	4	G♭	2700	5400	10800	21600	43200	86400	172800
		E♭	3240	6480	12960	25920	51840	103680	207360
	5	E♭♭	3375	6750	13500	27000	54000	108000	216000
	6	C♭	4050	8100	16200	32400	64800	129600	259200
Root 2025.	4	C♭	8100	16200	32400	64800	129600	259200	518400
		A♭	9720	19440	38880	77760	155520	311040	622080
	5	A♭♭	10125	20250	40500	81000	162000	324000	648000
	6	F♭	12150	24300	48600	97200	194400	388800	777600
Root 6075.	4	F♭	24300	48600	97200	194400	388800	777600	1555200
		D♭	29160	58320	116640	233280	466560	933120	1866240
	5	D♭♭	30375	60740	121480	242960	485920	971840	1943680
	6	B♭♭	36450	72900	145800	291600	583200	1166400	2332800

FIRST DIAGRAM. QUINTUPLE.

Root 25	4	Cb	500	1000	2000	4000	8000	16000
		Ab	600	1200	2400	4800	9600	19200
	5	Abb	625	1250	2500	5000	10000	20000
	6	Fb	750	1500	3000	6000	12000	24000
Root 37	4	Fb	1500	3000	6000	12000	24000	48000
		Db	1800	3600	7200	14400	28800	57600
	5	Dbb	1875	3750	7500	15000	30000	60000
	6	Bbb	2250	4500	9000	18000	36000	72000
Root 1125	4	Bbb	4500	9000	18000	36000	72000	144000
		Gb	5400	10800	21600	43200	86400	172800
	5	Gbb	5625	11250	22500	45000	90000	180000
	6	Ebb	6750	13500	27000	54000	108000	216000
Root 3375	4	Ebb	13500	27000	54000	108000	216000	432000
		Cb	16200	32400	64800	129600	259200	518400
	5	Cbb	16875	33750	67500	135000	270000	540000
	6	Abb	20250	40500	81000	162000	324000	648000
Root 10125	4	Abb	40500	81000	162000	324000	648000	1296000
		Fb	48600	97200	194400	388800	777600	1555200
	5	Fbb	50625	101250	202500	405000	810000	1620000
	6	Dbb	60750	121500	243000	486000	972000	1944000

SECOND DIAGRAM.
QUINTUPLE.

Root 1215.	6	D*	7290	14580	29160	58320	116640	233280	466560	933120
	5	B*	6075	12150	24300	48600	97200	194400	388800	777600
		B _h	5832	11664	23328	46656	93312	186624	373248	746496
	4	G*	4860	9720	19440	38880	77760	155420	310840	621680
Root 405.	6	G*	2430	4860	9720	19440	38880	77760	155520	311040
	5	E*	2025	4050	8100	16200	32400	64800	129600	259200
		E _h	1944	3888	7776	15552	31104	62208	124416	248832
	4	C*	1620	3240	6480	12960	25920	51840	103680	207360
Root 135.	6	C*	810	1260	3240	6480	12960	25920	51840	103680
	5	A*	675	1350	2700	5400	10800	21600	43200	86400
		A _h	648	1296	2592	5184	10368	20736	41472	82944
	4	F*	540	1080	2160	4320	8640	17280	34560	69120
Root 45.	6	F*	270	540	1080	2160	4320	8640	17280	34560
	5	D*	225	450	900	1800	3600	7200	14400	28800
		D _h	216	432	864	1728	3456	6912	13824	27648
	4	B	180	360	720	1440	2880	5760	11520	23040
Root 15.	6	B	90	180	360	720	1440	2880	5760	11520
	5	G*	75	150	300	600	1200	2400	4800	9600
		G _h	72	144	288	576	1152	2304	4608	9216
	4	E	60	120	240	480	960	1920	3840	7680
Root 5.	6	E	30	60	120	240	480	960	1920	3840
	5	C*	25	50	100	200	400	800	1600	3200
		C _h	24	48	96	192	384	768	1536	3072
	4	A	20	40	80	160	320	640	1280	2560

SECOND DIAGRAM. QUINTUPLE.

PlateXVI

Root 6075.	6	F##	36450	72900	145800	291600	583200	1166400	2332800
	5	D##	30375	60750	121500	24300	486000	972000	1944000
		D#	29160	58320	116640	233280	466560	933120	1866240
	4	B#	24300	48600	97200	194400	388800	777600	1555200
Root 2025.	6	B#	12150	24300	48600	97200	194400	388800	777600
	5	G##	10125	20250	40500	81000	162000	324000	648000
		G#	9720	19440	38880	77760	155520	311040	622080
	4	E#	8100	16200	32400	64800	129600	259200	518400
Root 675.	6	E#	4050	8100	16200	32400	64800	129600	259200
	5	C##	3375	6750	13500	27000	54000	108000	216000
		C#	3240	6480	12960	25920	51840	103680	207360
	4	A#	2700	5400	10800	21600	43200	86400	172800
Root 225.	6	A#	1350	2700	5400	10800	21600	43200	86400
	5	F##	1125	2250	4500	9000	18000	36000	72000
		F#	1080	2160	4320	8640	17280	34560	69120
	4	D#	900	1800	3600	7200	14400	28800	57600
Root 75.	6	D#	450	900	1800	3600	7200	14400	28800
	5	B#	375	750	1500	3000	6000	12000	24000
		Bb	360	720	1440	2880	5760	11520	23040
	4	G#	300	600	1200	2400	4800	9600	19200
Root 25.	6	G#	150	300	600	1200	2400	4800	9600
	5	E#	125	250	500	1000	2000	4000	8000
		Eb	120	240	480	960	1920	3840	7680
	4	C#	100	200	400	800	1600	3200	6400

SECOND DIAGRAM. QUINTUPLE.

Root 10125.	6	D##	60750	121500	243000	486000	972000	1944000
	5	B##	50625	101250	202500	405000	810000	1620000
		B#	48600	97200	194400	388800	777600	1555200
	4	G##	40500	81000	162000	324000	648000	1296000
Root 3375.	6	G##	20250	40500	81000	162000	324000	648000
	5	E##	16875	33750	67500	135000	270000	540000
		E#	16200	32400	64800	129600	259200	518400
	4	C##	13500	27000	54000	108000	216000	432000
Root 1125.	6	C##	6750	13500	27000	54000	108000	216000
	5	A##	5625	11250	22500	45000	90000	180000
		A#	5400	10800	21600	43200	86400	172800
	4	F##	4500	9000	18000	36000	72000	144000
Root 375.	6	F##	2250	4500	9000	18000	36000	72000
	5	D##	1875	3750	7500	15000	30000	60000
		D#	1800	3600	7200	14400	28800	57600
	4	B#	1500	3000	6000	12000	24000	48000
Root 125.	6	B#	750	1500	3000	6000	12000	24000
	5	G##	625	1250	2500	5000	10000	20000
		G#	600	1200	2400	4800	9600	19200
	4	E#	500	1000	2000	4000	8000	16000

Plate XVIII. **DIAGRAMS OF GAUDENTIUS.**

FIRST DIAGRAM.

SECOND DIAGRAM.

		576	<i>Nete hyperbolæon</i>	2592		
9	8				9	8
		648	<i>Hyperbolæon diatonos</i>	2304		
9	8				9	8
256	243	729	<i>Trite hyperbolæon</i>	2048	256	243
		768	<i>Nete diezeugmenon</i>	1944		
9	8				9	8
		864	<i>Diezeugmenon diatonos</i>	1728		
9	8				9	8
256	243	972	<i>Trite diezeugmenon</i>	1536	256	243
		1024	<i>Paramesos</i>	1458		
9	8				9	8
		1152	<i>Meson</i>	1296		
9	8				9	8
		1296	<i>Meson diatonos</i>	1152		
9	8				9	8
256	243	1458	<i>Parhypate meson</i>	1024	256	243
		1536	<i>Hypate meson</i>	972		
9	8				9	8
		1728	<i>Hypaton diatonos</i>	864		
9	8				9	8
256	243	1944	<i>Parhypate hypaton</i>	768	256	243
		2048	<i>Hypate hypaton</i>	729		
9	8				9	8
		2304	<i>Proslambanomencs</i>	648		

Inversion of the Scales.

First Diagram.

First Diagram: Musical notation for the first inversion of the scale. The staff shows the notes A, G, F, E, D, C, B, A, G, F, E, D, C, B, A. Below the staff are two rows of intervals and a row of numbers. The intervals are 8/9, 8/9, 2 4 3 / 2 5 6, 8/9, 8/9, 2 4 3 / 2 5 6, 8/9, 8/9, 8/9, 2 4 3 / 2 5 6, 8/9, 8/9, 2 4 3 / 2 5 6, 8/9. The numbers are 576, 648, 729, 768, 864, 972, 1024, 1152, 1296, 1458, 1536, 1728, 1944, 2048, 2304, 2592.

Second Diagram.

Inversion of the Scales.

FIRST DIAGRAM.

Roots, & their Octaves.

Roots Harmonics

1	B	4					1024	2048	
	G	5							
	E	6							
3	E	12				768	1536		
	C	15							
	A	18							
9	A	36				1152	2304		
	F	45							
	D	54							
27	D	108			864	1728			
	B \flat	135							
	G	162							
51	G	324		648	1296	2592			
	E \flat	405							
	C	486							
243	C	972		972	1944				
	A \flat	1215							
	F	1458							
729	F	2916	729	1458					
	D \flat	3645							
	B \flat	4374							

SECOND DIAGRAM. Roots and their Octaves.

PlateXXI

Roots	Harmonics							
	F#	4374						
	D#	3645						
729	B	2916	729	1458				
	B	1458						
	G#	1215						
243	E	972		972	1944			
	E	486						
	C#	405						
81	A	324		648	1296	2592		
	A	162						
	F#	135						
27	D	108			864	1728		
	D	54						
	B	45						
9	G	36				1152	2304	
	G	18						
	E	15						
3	C	12				768	1536	
	C	6						
	A	5						
1	F	4					1024	2048

Tetrachord first Diagram. Tetrachord second Diagram

fig:1

	$\frac{16}{15}$	$\frac{10}{9}$	$\frac{9}{8}$	
B	C	D	E	
3 2	3 0	2 7	2 4	
G	A	A	A	
4 0	3 6	3 6	3 6	
E	E	F	E	
4 8	4 8	4 5	4 8	
E	A	D	A	
9 6	7 2	5 4	7 2	
Roots	1	3	9	3

fig:2

	$\frac{15}{16}$	$\frac{9}{10}$	$\frac{8}{9}$	
F	E	D	C	
6 4	6 0	5 4	4 8	
C	C	B	C	
4 8	4 8	4 5	4 8	
A	G	G	G	
4 0	3 6	3 6	3 6	
F	C	G	C	
3 2	2 4	1 8	2 4	
Roots	1	3	9	3

The Scale, or Diatonic Octave in the first Diagram.

fig:3

	$\frac{9}{10}$	G	$\frac{8}{9}$	F	$\frac{15}{16}$	E	$\frac{8}{9}$	D	$\frac{9}{10}$	C	$\frac{15}{16}$	B	$\frac{8}{9}$	A
	3 6	4 0	4 5	4 8	5 4	6 0	6 4	7 2						
	$\frac{2}{1}$	$\frac{6}{5}$	$\frac{6}{5}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{6}{5}$	$\frac{3}{2}$	$\frac{2}{1}$						
F.B.	7 2. A	4 8. E	5 4. D	7 2. A	10 8. D	7 2. A	9 6. E	14 4. A						
Roots	3	1	9	3	9	3	1	3						

The Scale, or Diatonic Octave, in the Second Diagram.

fig:4

	$\frac{9}{8}$	D	$\frac{10}{9}$	E	$\frac{16}{15}$	F	$\frac{9}{8}$	G	$\frac{10}{9}$	A	$\frac{9}{8}$	B	$\frac{16}{15}$	C
	2 4	2 7	3 0	3 2	3 6	4 0	4 5	4 8						
	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{2}{1}$						
F.B.	12. C	18. G	24. C	16. F	24. C	32. F	36. G	24. C						
Roots	3	9	3	1	3	1	9	3						

FIRST DIAGRAM.
TETRACHORDS CONJOINED.

FIG. 1.

ANTECEDENT.				MEAN.			CONSEQUENT.			
$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$				$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$			$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$			
B	C	D	E	F	G	A	B \flat	C	D	
512	480	432	384	360	324	288	270	243	216	
$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	
768	576	864	576	432	648	432	324	486	324	
FUNDAMENTAL BASES	E	A	D	A	D	G	D	G	C	G
ROOTS.	1	3	9	3	9	27	9	27	81	27

FIRST DIAGRAM.
TETRACHORDS DISJOINED.

FIG.3.

$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$				$\frac{1}{1}$	$\frac{9}{8}$	$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$				$\frac{1}{1}$	$\frac{9}{8}$	$\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$			
A	B \flat	C	D	D	E	F	G	A	A	B	C	D	E		
288	270	243	216	216	192	180	162	144	144	128	120	108	96		
$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$		
432	324	486	324	432	288	216	324	216	288	192	144	216	144		
FB. R.	D 9	G 27	C 81	G 27	D 9	A 3	D 9	G 27	D 9	A 3	E 1	A 3	D 9	A 3	

SECOND DIAGRAM.

Plate XXIV.

TETRACHORDS CONJOINED.

FIG. 2.

ANTECEDENT.			MEAN.			CONSEQUENT.				
$\frac{15}{16}$	$\frac{9}{10}$	$\frac{8}{9}$	$\frac{15}{16}$	$\frac{9}{10}$	$\frac{8}{9}$	$\frac{15}{16}$	$\frac{9}{10}$	$\frac{8}{9}$		
F	E	D	C	B	A	G	F#	E	D	
512	480	432	384	360	324	288	270	243	216	
$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	
256	384	288	192	288	216	144	216	162	108	
FUNDAMENTAL BASSES	F	C	G	C	G	D	G	D	A	D
ROOTS	1	3	9	3	9	27	9	27	81	27

SECOND DIAGRAM.

TETRACHORDS DISJOINED.

FIG. 4.

$\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ $\frac{1}{1}$ $\frac{8}{9}$					$\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ $\frac{1}{1}$ $\frac{8}{9}$					$\frac{15}{16}$ $\frac{9}{10}$ $\frac{8}{9}$ $\frac{1}{1}$ $\frac{8}{9}$				
G	F#	E	D	D	C	B	A	G	G	F	E	D	C	
288	270	243	216	216	192	180	162	144	144	128	120	108	96	
$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{2}{1}$	
144	216	162	108	144	96	144	108	72	96	64	96	72	48	
F.B.	G	D	A	D	G	C	G	D	G	C	F	C	G	C
R.	9	27	81	27	9	3	9	27	9	3	1	3	9	3

FIRST DIAGRAM. QUINTUPLE.

*Communication between the
minor and major Scales.*

1 st Sect. Root 1.			2 ^d Sect. Root 5.		3 ^d Sect. Root 25.		4 th Sect. Root 125.	
Root 1.	B	16	G. 20		Eb. 100			
	G	20						
	E	24	E 24	Eb 25				
Root 3.	E	48	C 30		C. 120. Cb. 125		Cb. 500.	
	C	60	C 60		Ab. 150		Ab 600. Abb. 625	
	A	72	A 72	Ab 75	Ab. 300		Fb. 750	
Root 9.	A	144	F 90		F. 360. Fb. 375		Fb 1500	
	F	180	F 180		Db. 450		Db 1800. Dbb 1875	
	D	216	D 216	D 225	Db. 900		Bbb 2250.	
Root 27.	D	432	Bb 270		Bb. 1050. Bbb. 1125		Bbb. 4500	
	Bb	540	Bb 540		Gb. 1350		Gb. 5400. Gbb 5625	
	G	648	G 648	Gb 675	Gb. 2700		Eb. 6750	
Root 81.	G	1296	Eb. 810		Eb. 3240. Ebb. 3375		Ebb. 13500	
	Eb	1620	Eb 1620		Cb. 4050		Cb. 16200. Cbb 16875	
	C	1944	C 1944	Cb. 2025	Cb. 8100		Abb 20250	
Root 243.	C	3888	Ab 2430		Ab. 9720. Abb. 10125		Abb. 40500	
	Ab	4860	Ab 4860		Fb. 12150		Fb 48600. Fbb 50625	
	F	5832	F 5832	Fb 6075	Fb. 24300		Dbb. 60750	
Root 729.	F	11664	Db. 7290		Db. 29160. Dbb 30375			
	Db	14580	Db 14580		Bbb. 36450			
	Bb	17496	Bb 17496	Bbb 18225				
			Gb 21870					

Communication between the
minor and major Scale

SECOND DIAGRAM. QUINTUPLE.

*Communication between the
minor and major Scales.*

Root 729.	F #	17496	F # 17496	F # 18225	A # 21870
	D #	14580	D # 14580	F # 36450	
	B	11664	D # 7290	D # 29160	D # 30375
Root 243.	B	5832	B 5832	B # 6075	B # 24300
	G #	4860	G # 4860	B # 12150	D # 60750
	E	3888	G # 2430	G # 9720	B # 48600
Root 81.	E	1944	E 1944	E # 2025	B # 50625
	C #	1620	C # 1620	E # 8100	G # 40500
	A	1296	C # 810	E # 4050	G # 20250
Root 27.	A	648	A 648	A # 675	E # 16200
	F #	540	F # 540	C # 3240	E # 16875
	D	432	F # 270	C # 3375	C # 13500
Root 9.	D	216	D 216	A # 2700	C # 6750
	B	180	B 180	A # 1350	A # 5400
	G	144	B 90	F 1080	A # 5625
Root 3.	G	72	G 72	F # 1125	F # 4500
	E	60	G # 75	D # 900	F # 2250
	C	48	E 60	D # 450	D # 1800
Root 1.	C	24	C 24	B 360	D # 1875
	A	20	E 30	B # 375	B # 1500
	F	16	C # 25	G # 300	B # 750
Root 1. 1 st Sect.			Root 5. 2 nd Sect.		
			Root 25. 3 rd Sect.		
			Root 125. 4 th Sect.		

The Three Terms, or fundamental Bases, proper to form three Scales with Minor 3^{ds} & three with Major 3^{ds}

1 Flat.	D	{	A	3	3	{	30. C	}	F.	1 Flat.
			D				90. F			
			G				270. B \flat			
2 Flats.	G	{	D	9	3	{	90. F	}	B \flat .	2 Flats.
			G				270. B \flat			
			C				810. E \flat			
3 Flats.	C	{	G	27	3	{	270. B \flat	}	E \flat .	3 Flats.
			C				810. E \flat			
			F				2430. A \flat			

Fig. 2.

The Three Terms, or fundamental Bases, proper to form three Scales with Major 3^{ds} & three with Minor 3^{ds}

3 Sharps.	A	{	E	243	3	{	1620 C \sharp	}	F \sharp .	3 Sharps.
			A				540 F \sharp			
			D				180 B			
2 Sharps.	D	{	A	81	3	{	540 F \sharp	}	B.	2 Sharps.
			D				180 B			
			G				60 E			
1 Sharp.	G	{	D	27	3	{	180 B	}	E.	1 Sharp.
			G				60 E			
			C				20 A			

FIRST DIAGRAM.

Illustration of the Pythagorean Numbers.

Roots	Pythagorean Numbers	Suspension of 4 th	Suspension of 9 th	At 4 th Term compounded.	At 4 th Term double compounded.
	6	E	E 9 th		F 9 th
		C		D 7 th	D 7 th
Base		A	A 8 th	A 5 th	
	8	B		B 5 th	B 5 th
		G		G 3 ^d	G 3 ^d
Base		E	E 5 th	E 1	E 1
	9	A			
		F	F 3 ^d		
Base		D	D 4 th	D 1	
	12	E			
		C			
Base		A	A 1		

SECOND DIAGRAM.

Illustration of the Pythagorean Numbers.

	Roots		Suspension of 1/2	Suspension of 1/4 9	A third Term compounded	A third Term double compounded
3	12	G E C	C 8 th	G 9 th C 5 th	F b 7 th	9 th A b orb F b 7 th
9	9	D B G	G 5 th	D 5 th B 3 ^d # G 1.	D 5 th B 3 ^d # G 1.	D 5 th B 3 ^d # G 1.
1	8	C A F	F 4 th	A 3 rd F 1.		
3	6	G E C	C 1.			

*Pythagorean
Numbers.*